DISCRETE MULTIVARIABLE PROCESS CONTROL

By

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A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

1986

To Diamond

ACKNOWLEDGEMENTS

I wish to express my gratitude to my advisor, Spyros A. Svoronos, for his support, which made this and other research possible during my years at the University of Florida, 1982 to 1986. I am grateful to him for suggesting this topic and for guiding the research effort during all the stages of the development of this project, while allowing my endeavors in other research areas with Professor G. Lyberatos. Dr. Lyberatos is thanked fullheartedly for introducing me to the nonlinear systems in his own inspiring way. My gratitude is also expressed to Professor L. E. Johns, who has been a persistent source of inspiration, guidance, and enlightening suggestions since April, 1983. Professor V. M. Popov, from the Department of Mathematics, is thanked for many helpful discussions and for inspiring teaching. Professor E. W. Kamen, from the Department of Electrical Engineering, is also acknowledged for having an important influence through teaching and discussions on many parts of this work. Professor H. Lee is thanked for being a member of my graduate committee. Professor J. P. O'Connell is thanked for having an indirect but essential influence on most of what I completed at the University of Florida. Finally, I would like to thank my colleague, A. Papadoulis, for many valuable discussions, and Teresa Agrillo, for her excellent typing.

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Abstract of Doctoral Dissertation Presented to the Graduate
School of the University of Florida in Partial Fulfillment of
the Requirements for the Degree of Doctor of Philosophy

DISCRETE MULTIVARIABLE PROCESS CONTROL

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December, 1986

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Major Department: Chemical Engineering

The problem of digital control of multivariable processes with multiple delays and inverse response characteristics is addressed. The effect of the delays on the parametrization of linear discrete-time models for multivariable adaptive controller design is studied. The inversion of multivariable plants for adaptive and internal model control is addressed via factorization of the process transfer matrix. Finally, the asymptotic stability of multivariable plants controlled by self-tuning controllers is investigated.

Self-tuning control algorithms based on the minimization of a quadratic performance measure are developed for processes with potentially differing measurement and control delays. Algorithms with different computational requirements depending upon the delay structure of the process are presented. A method for factorizing the process transfer matrix is given, which requires a priori knowledge of only the process delays and which allows for self-tuning controller design in the presence of arbitrary input-output delays. The left interactor matrix is used for determining the invertible part of plants with arbitrary delays and no finite unstable zeros. The invertible part is used in designing the optimal internal model controller for such processes. The right interactor matrix is introduced and used for adaptive controller and feedforward estimator design for delay processes. The notion of the generalized interactors is introduced for processes with multiple delays, multiple unstable zeros and non-trivial cross-couplings. The generalized interactors contain all the noninvertible elements of the process transfer matrix which are invariant under stable feedback control. The left generalized interactor leads to the best possible internal model controller and in a dual way the right generalized interactor leads to the best possible feedforward disturbance estimator. Finally, the asymptotic stability of quadratic self-tuning controllers is established. It is shown that for any initial state of the plant and/or the estimator and for suitable offline choices of the weights in the performance measure, the input and output deviations tend to zero while all system variables remain bounded.

CHAPTER I
INTRODUCTION

Process control is used by the chemical industries to ensure safe operation of the plant, to suppress the influence of external disturbances, and to optimize the plant's performance. Process control is a necessary part of competitive plants due to: a) the increasingly complex structure of chemical processes due to more efficient management of energy and raw materials b) the high and consistent quality specifications which the final product should meet; and c) the various environmental and operational constraints which the plant units must satisfy.

Modern control theory now has a number of valuable tools at the disposal of the control engineer. Many of the modern control techniques are implemented industrially by digital computers and microprocessors. The results of this work concern digital control and especially discrete adaptive and internal model control. Both are particularly useful in chemical process applications.

The idea for a controller which could adapt itself automatically to changing characteristics of the controlled process first originated in aerospace problems in which classical linear control performed unsatisfactorily due to significant in-flight variation of the response characteristics of the process within the flight envelope. Classical linear control is mainly based on a model of the system and its environment obtained off-line by some system identification technique from practical experiments. In the case of a changing process, at most three controller parameters can be adjusted manually during operation.

As far as industrial plants are concerned, the complexity of the processes, the absence of reliable models, the essential nonlinearity of the processes, and the often changing operating conditions make self-tuning control seem a promising alternative to classical linear control. Self-tuning controllers (STCs) are discrete time adaptive control algorithms. They were introduced by Åström and Wittenmark (1) in 1973. Since then, the original algorithm has been improved and today's STCs feature design simplicity together with the ability to handle open loop unstable or inverse response systems. The part of this work which deals with adaptive control is concerned with STCs which are performance oriented, i.e., the control law is designed to optimize an a priori chosen quadratic performance index. Such algorithms are particularly useful in applications where the system is highly nonlinear, since they are likely to be more robust than other self-tuning algorithms (2). Also, STCs based on the minimization of a quadratic performance index which includes penalization of both output and input deviations will give improved economic performance of the controlled plant, especially when the control is not inexpensive. Finally, such STCs can control open loop unstable or inverse response systems, provided that the weights in the performance criterion are suitably chosen. This makes them useful for digital control since most practical processes, particularly when digitally controlled, are likely to exhibit inverse response characteristics (3). Chapters II, III, and IV present multivariable quadratic STCs for systems with multiple input-output delays. Chapter II is application oriented. The design and computational features of some quadratic STCs given and the effect of multiple delays are pointed out. An application to a continuous stirred tank reactor is also given. The part of Chapter III which deals with STCs presents two methods for transforming the original model form to a form suitable for "quadratic" STC design. Chapter IV establishes the stability of quadratic STCs.

The Internal Model Principle (IMP) was introduced by Francis and Wonham (4) and addresses the problem of tracking a desired output signal in spite of the presence of disturbances, something very desirable in applications. According to the IMP, in order to achieve perfect tracking, knowledge of the dynamics of both the desired output signal and the disturbance is required and the modes of both must be included in the controller design.

The Internal Model Control (IMC) configuration of Brosilow (5) and Garcia and Morari (6) deals with the problem of set point tracking despite disturbances in the measurement. The IMC configuration allows for obtaining the optimal controller, i.e., the one which achieves optimal output-set-point matching. It also allows for an assessment of robustness in case of model/plant mismatch. Having the optimal feedback controller for each alternative design allows for optimal plant-controller design to meet desired closed loop response characteristics (7). The IMC design requires the plant to be open loop stable.

In this work, the IMC design is linked to some structural characteristics of the plant which dictate the optimal way that an output signal can be tracked despite the measurement disturbances.

Chapter III gives a method for obtaining the best possible IMC controller for multivariable plants with multiple delays. The method is based on the interactor matrix concept which was introduced by Wolowich and Falb (8). A right interactor matrix is introduced here and used in adaptive controller design and in the design of an optimal feedforward disturbance estimator. The generalized interactor matrices introduced in Chapter III allow for optimal IMC controller and feedforward estimator design for multivariable processes exhibiting multiple delays, multiple unstable zeros, and nontrivial cross-couplings.

CHAPTER II
EFFECT OF DELAYS ON THE PARAMETRIZATION OF LINEAR MODELS FOR ADAPTIVE CONTROLLERS

2.1 Introduction

Self-tuning controllers (STCs) are adaptive control algorithms in which the plant parameters are estimated online and consequently used in the determination of the control law. The STCs do not require a detailed plant model before start up and can adapt to slow changes of the process.

The control law of a multivariable STC can be designed on the basis of pole/zero placement consideration (e.g., Wellstead et al. (9), Åström and Wittenmark (10), Prager and Wellstead (11), Vogel and Edgar (12)), as minimum variance (e.g., Åström and Wittenmark (1), Keviczky et al. (13), Borisson (14), Goodwin et al. (15), Bücholt and Kümmel (16)), or on the basis of a more general quadratic performance measure that penalizes input deviation and/or input velocity in addition to measurement deviations. STCs of the latter type are particularly valuable in applications where control energy is expensive or where it is desired to limit excessive control valve movement. Several such algorithms have been developed (e.g., Koivo (17), Keviczky and Kumar (18), Bayoumi et al. (19)), but they all require that inputs and outputs are subject to the same time delay.

However, in a plant there are several control variables and in the general case each one has a distinct time delay. For example, in a reactor each one of two possible control variables, inlet reactants' flow rate and inlet coolant temperature, affects the process subject to its own delay. Similarly, in a plant each one of the measurement variables has a distinct delay. For instance, in the above considered perfectly mixed reactor, two measurement variables could be outlet temperature and concentration and in general the concentration measurement devices introduce greater delays than the temperature measurement devices (thermocouples). Thus, each measurement variable is again subject to a distinct delay. The individual time delays of measurements and controls are very often encountered in practice and result in poor performance of conventional P.I.D. controllers. The present chapter develops adaptive STC algorithms which deal most effectively with such cases.

Multivariable Self-Tuning Controllers Applicable to Systems with Measurements of Differing Time Delays

Discrete multiple-input multiple-output linear systems with measurements of differing time delays are considered here. By suitable manipulations, the original system model is rewritten in three alternative forms, each of which leads to a direct calculation of the optimal control policy. By combining a recursive least squares parameter estimator with the estimated model optimal control law for each model form, three alternative self-tuning control schemes are obtained. Each one uses a different amount of the most recently obtained information and involves different computational requirements. Which one is preferable depends on the number of inputs, number of outputs, system order, and time delays of the process to be controlled, as well as on the capabilities of the available computer. All three self-tuning control algorithms are shown to be successful in controlling a continuous-flow stirred tank reactor model.

The Model

The process to be controlled is assumed to be described adequately by the following linear model of order n+1:

$$A(q^{-1})z(t+1) = s+B(q^{-1})u(t) \tag{2-1}$$

where $A(q^{-1})$, $B(q^{-1})$ are matrix polynomials in the backward shift operator q^{-1}, i.e., $q^{-1}y(t) = y(t-1)$,

$$A(q^{-1}) = I-q^{-1}\sum_{i=1}^{n} A^{(i)}q^{-i}$$

$$B(q^{-1}) = \sum_{i=0}^{n} B^{(i)}q^{-i}$$

and where $u(t)$ is the m-dimensional control vector and $z(t)$ is the ℓ-dimensional state vector at time t. Also s is an ℓ-dimensional parameter vector; its inclusion allows the consideration of set point changes and of nonzero mean load changes. The states can be measured, but the measurements may be subject to delays. The process parameters are the unknown matrices: s, $A^{(i)}$, $B^{(i)}$. These may change whenever the process undergoes load or set point changes.

Let y_i denote the measurement of z_i with a time delay k_i. Then,

$$z_i(t) = y_i(t+k_i) \qquad i=1,2\ldots\ell \qquad (2\text{-}2)$$

Without loss of generality, it can be assumed that the outputs have been ordered so that

$$k_1 \leqslant k_2 \quad \ldots \leqslant k_\ell = k$$

Let ´ denote transpose. Calling

$$w'(t) = [y_1(t+k_1-k),\ y_2(t+k_2-k),\ldots,\ y_\ell(t)]$$

model (2-1) is rewritten as

$$w(t+k+1) = s+A_0(q^{-1})w(t+k)+B(q^{-1})u(t) \qquad (2\text{-}3)$$

where

$$A_0(q^{-1}) = q[A(0)-A(q^{-1})] = \sum_{i=1}^{n} A^{(i)}q^{-i}$$

If in equation (2-3) $w(t+k-1),\ldots,w(t+1)$ are substituted from this same equation, the result is the predictor of $w(t+k+1)/t$:

$$w(t+k+1) = s^*+C(q^{-1})w(t)+D(q^{-1})u(t) \qquad (2\text{-}4)$$

where

$$s^* = R_{k+1}s$$

$$C(q^{-1}) = \sum_{i=0}^{n} C_{k+1}^{(i)}q^{-i} \qquad (2\text{-}5)$$

$$D(q^{-1}) = \sum_{i=0}^{n} D_{k+1}^{(i)} q^{-i}$$

and the parameters are given by the matrix discrete convolution equations:

$$R_{k+1} = I + \sum_{j=0}^{k-1} A^{(j)} R_{k-j}$$

$$C_{k+1}^{(i)} = A^{(k+i)} + \sum_{j=0}^{k-1} A^{(j)} C_{k-j}^{(i)} \qquad i=0,1,2,\ldots n \tag{2-6}$$

$$D_{k+1}^{(i)} = B^{(i)} + \sum_{j=0}^{k-1} A^{(j)} D_{k-j}^{(i-j-1)} \qquad i=0,1,\ldots n+k$$

Here a superscript of less than zero signifies a zero matrix. Also zero are $A^{(j)}$ and $B^{(j)}$ for $j > n$.

Model Form I

From (2-4) it follows that

$$y_j(t) = x_j'(t-k_j-1)\ \theta_j \qquad j=1,2,\ldots \ell \tag{2-7}$$

where

$$x_j'(t-k_j-1) = [1\ w'(t-k_j-1)\ldots w'(t-k_j-1-n)$$
$$u'(t-k_j-1)\ldots u'(t-k_j-k-1-n)]$$

$$\theta_j' = [s_j^*\ C_j^{(0)}\ldots C_j^{(n)}\ D_j^{(0)}\ldots D_j^{(n+k)}]$$

where $C_j^{(i)}$, $D_j^{(i)}$ denote jth row.

Model Form II

Shifting of each of the equations in (2-7) by k_j-k yields the following form:

$$y_j(t+k_j-k) = x_\ell'(t-k-1)\ \theta_j \qquad j=1,2,\ldots,\ell \tag{2-8}$$

Model Form III

From model (2-1), we obtain

$$\det A(q^{-1})z(t+1) = \operatorname{adj}(A(q^{-1}))[s+B(q^{-1})u(t)] \tag{2-9}$$

where det stands for determinant and adj for adjoint. In this form we see that each output depends merely on its own past values and on the inputs.

From (2-9), we get

$$w(t+k+1) = s^{**}+F(q^{-1})w(t)+H(q^{-1})u(t) \tag{2-10}$$

where s** is again an ℓ-dimensional parameter vector and

$$F(q^{-1}) = \operatorname{diag}[f_{ii}(q^{-1})];\ f_{ii}(q^{-1}) = \sum_{r=0}^{d} f_{rii}q^{-r}$$

$$(H(q^{-1}))_{ij} = h_{ij}(q^{-1});\ h_{ij}(q^{-1}) = \sum_{r=0}^{d+k_i} h_{rij}q^{-r}$$

$$d = \ell(n+1)-1$$

From equation (2-10), it follows as previously:

$$y_i(t) = x_i'(t-k_i-1)\ \theta_i \tag{2-11}$$

where

$$x_i'(t-k_i-1) = [1\ y_i(t-k_i-1) \ldots y_i(t-k_i-d-1)$$
$$u'(t-k_i-1) \ldots u'(t-2k_i-d-1)]$$

and

$$\theta_i' = [s_i^{**}\ f_{oii} \cdots f_{dii}\ h_{oi1} \cdots h_{oim} \cdots h_{(d+k_i)i1} \cdots h_{(d+k_i)im}]$$

Any of the parameter estimators given in Appendix A can be used to update the parameter vectors θ_i in (2-7), (2-8) and (2-11), if the input and output data up to and including time t are available.

The Control Law

Let $e(t) = w(t)-y_s$

$\bar{u}(t) = u(t)-u_s$

$\tilde{u}(t) = u(t)-u(t-1)$

where y_s is the set-point and u_s is the corresponding steady state control value. The performance measure is chosen as

$$J(u(t)) = e'(t+k+1)Q_1e(t+k+1) + \bar{u}'(t)Q_2\bar{u}(t)+\tilde{u}'(t)Q_3\tilde{u}(t)$$

The weight matrix Q_1 is positive definite while Q_2 and Q_3 are positive semidefinite. The last term enables penalizing sudden jumps in the control action. If the input value u_s is unknown, then $Q_2 = 0$. If $J(u(t))$ is minimized with respect to $u(t)$ and the estimated parameters are substituted for the true ones according to the certainty equivalence principle (Bar-Shalom and Tse (20)), one obtains the following control laws.

For model (2-4) and estimation via model forms (2-7) or (2-8):

$$u(t) = -[\hat{D}_t^{(0)\prime} Q_1 \hat{D}_t^{(0)} + Q_2 + Q_3]^{-1} [\hat{D}_t^{(0)\prime} Q_1 a(t) - Q_2 u_s - Q_3 u(t-1)] \tag{2-12}$$

where ´ denotes transpose and

$$a(t) = \hat{s}_t^{*} + \hat{C}_t(q^{-1}) w(t) + \sum_{i=1}^{n+k} \hat{D}_t^{(i)} u(t-1)$$

and where $\hat{\cdot}_t$ denotes estimate at time t. The subscript k+1 in the matrices $\hat{D}_t^{(i)}$ was dropped for notational convenience.

For model (2-10) and estimation via model form (2-11):

$$u(t) = -[\hat{H}_t^{(0)\prime} Q_1 \hat{H}_t^{(0)} + Q_2 + Q_3]^{-1} [\hat{H}_t^{(0)} Q_1 b(t) - Q_2 u_s - Q_3 u(t-1)] \tag{2-13}$$

with

$$\hat{H}_t^{(0)} = \hat{H}_t(0)$$

and

$$b(t) = \hat{s}_t^{**} + \hat{F}_t(q^{-1}) w(t) + \sum_{i=1}^{n+k} \hat{H}_t^{(i)} u(t-i)$$

The three STCs resulting from the three model forms, (2-7), (2-8), and (2-11), and the associated control laws, (2-12), (2-12), and (2-13), are called "standard," "simplified," and "decoupled," respectively. Some computational characteristics of these are given in Table 2.1. The decoupled STC has the advantage that it uses all the recent information available for the control law determination while the other two STCs do not. All three were tested successfully in the control of a CSTR reactor. In this application, the states, temperature and concentration, were measured with 0 and 4 minutes time delay.

Table 2.1
Characteristics of the "standard," "simplified," and "decoupled" STCs.

Algorithm	Uses information on $z(\tau)$ for $\tau > k_\ell$ for the parameter estimate updating	Uses information on $z(\tau)$ for $\tau > k_\ell$ for the control policy determination	Total number of estimated parameters	Total number of elements of the covariance matrices (matrix)
"simplified"	no	no	$\ell[1+(n+1)\ell]+ \ell m(n+1)+ \ell m k_\ell$	$\{[1+(n+1)\ell]+ m(n+1)+ m k_\ell\}^2$
"standard"	yes	no	$\ell[1+(n+1)\ell]+ \ell m(n+1)+ \ell m k_\ell$	$\ell\{[1+(n+1)\ell]+ m(n+1)+ m k_\ell\}^2$
"decoupled"	yes	yes	$\ell[1+(n+1)\ell]+ \ell^2 m(n+1)+ \sum_{i=1}^{\ell} k_i$	$\sum_{i=1}^{\ell}\{1+(n+1)\}+ m[\ell(n+1)+k_i]\}^2$

A CSTR with Measurement Delays

Consider an irreversible, first order exothermic reaction carried out in a perfectly mixed continuous-flow stirred tank reactor (CSTR). Under the assumptions of constant densities, negligible heat losses, and constant holdup, the system is described by the following equations:

$$\frac{dc_A}{dt} = \frac{F}{V} c_{A_o} - \frac{F}{V} c_A - \kappa\ e^{-E/RT} c_A \qquad (2\text{-}14)$$

$$\frac{dT}{dt} = \frac{F}{V} T_o - \frac{F}{V} T + \left[\frac{-\Delta H}{\rho c_p}\right] \kappa\ e^{-E/RT} c_A - \frac{UA}{V\rho c_p} (T - T_c) \qquad (2\text{-}15)$$

The notation and parameter values are given below:

c_A: reactant concentration

T: reactor temperature

T_c: coolant temperature

F: flowrate

V: holdup (1 m^3)

c_{A_o}: feed concentration (8.0 x 10^3 gmol/m^3)

κ: Arrhenius factor (4.7 x 10^5 sec^{-1})

E: activation energy (5.0 x 10^4 J/gmol)

R: gas constant (8.31434 J/K·gmol)

T_o: feed temperature (300K)

ΔH: heat of reaction (-2.0 x 10^4 J/gmol)

ρ: density (800 kg/m^3)

c_p: specific heat (3.0 x 10^3 J/kg·K)

U: overall heat transfer coefficient (1.0 x 10^3 J/m^2·sec·K)

A: heat transfer area (25 m^2)

Let the flowrate F and the coolant temperature T_c be the control variables. Furthermore, let the temperature T be measured with negligible time delay and the reactant concentration c_A be measured with a time delay of 4 minutes. These measurements are not assumed noise-free.

When implementing STCs, it is advisable, for numerical reasons, that the outputs and inputs be scaled so that they are of approximately the same order of magnitude. For this reason, when applying the three previously developed STCs to the model of equations (2-14) and (2-15), the outputs z_1, z_2 and the inputs u_1, u_2 were taken to be

$$z_1 = \frac{T}{300^{\circ}K} \qquad u_1 = \frac{T_c}{300^{\circ}K}$$

$$z_2 = \frac{c_A}{8000\ \mathrm{gmol/m^3}} \qquad u_2 = \frac{F}{3.10^{-4}\mathrm{m^3/sec}}$$

A sampling interval of 2 minutes was chosen. Thus, the time delay parameters were $k_1 = 0$, $k_2 = 2$ and the outputs and corresponding measurements were related by

$$y_1(t) = z_1(t)+e_1(t) \qquad y_2(t) = z_2(t-2)+e_2(t)$$

where $e_1(t)$ and $e_2(t)$ denote Gaussian white noise terms. The remaining a priori known algorithm parameters were: $n = 0$ (system is first order), $m = 2$ (two control variables), and $\ell = 2$ (two measurements). Thus, from the third column of Table 2.1 it follows that all three algorithms had the same number of parameters, 18. However, as the last column of

Table 2.1 indicates, the "simplified" algorithm required somewhat fewer calculations than the "decoupled."

The performance measure weight matrices used were

$$Q_1 = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \quad Q_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad Q_3 = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.017 \end{bmatrix}$$

The concentration deviation from the set points ($y_2 - y_{2s}$) was weighted 10 times as much as the temperature deviation ($y_1 - y_{1s}$) because concentration is the output of main concern. No weight was given to the deviation of the inputs from their steady state values ($u - u_s$) because these steady state values (u_s) were considered unknown after set point changes. Finally, the input change ($u(t) - u(t-1)$) was weighted low, since large set point changes were to be made. The coolant temperature change was weighted three times the flowrate change because in practice it is harder to accomplish large changes in T_c than in F. The estimation scheme RLS-VFF2-MR given in Appendix A was used for the "standard" and the "decoupled" algorithms. The estimator RLS-VFF2 was used for the simplified algorithm.

The initial set points were

$z_1 = 1.1$ (T = 330K) $\quad$ $z_2 = 0.15$ (85% conversion)

At t = 400 min, the concentration set point was changed to

$$z_2 = 0.20$$

while the temperature set point was left unchanged ($z_1 = 1.1$).

Figures 2-1a to 2-1c and 2-2a to 2-2c show the response of the system to "standard" self-tuning control. The STC was started at t = 60 min. It was quite successful both in regulation and in following the set point change.

Even though noise was not accounted for in the development of the three algorithms, the STCs successfully filter it out. This can be seen by comparing Figure 2-1b (Reactor Temperature Measurement) to Figure 2-1c (Actual Reactor Temperature).

Figures 2-3a, 2-3b, 2-4a, and 2-4b present the performance of the "simplified" STC. Very small degradation of performance is observed.

Finally, Figures 2-5a, 2-5b, 2-6a, and 2-6b present u_1, z_1, u_2, and z_2 for the "decoupled" algorithm. Its performance is equivalent to that of the "standard" STC.

Conclusions

Three self-tuning controllers are developed for systems with measurements of differing time delays. The "decoupled" one utilizes every available piece of information for both the parameter updating and the control policy determination but may require a large number of on-line calculations if the number of measurements is large. The "standard" STC does not use part of the most recent information for the control policy determination but, for some systems, it may involve considerably fewer calculations than the "decoupled" algorithm and in many cases there is no appreciable deterioration of performance. By not using some of the most recently obtained

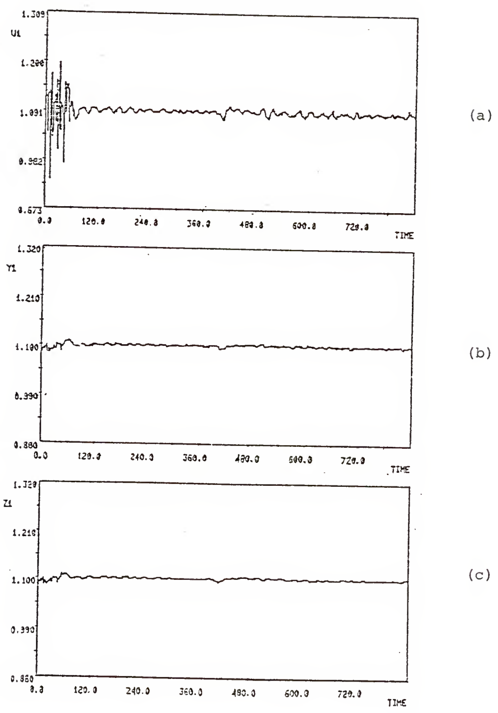

Figure 2-1. Response of the "standard" STC
(a) Coolant temperature (scaled) vs. time in min.
(b) Reactor temperature measurement (scaled) vs. time (min).
(c) Actual reactor temperature (scaled) vs. time (min).

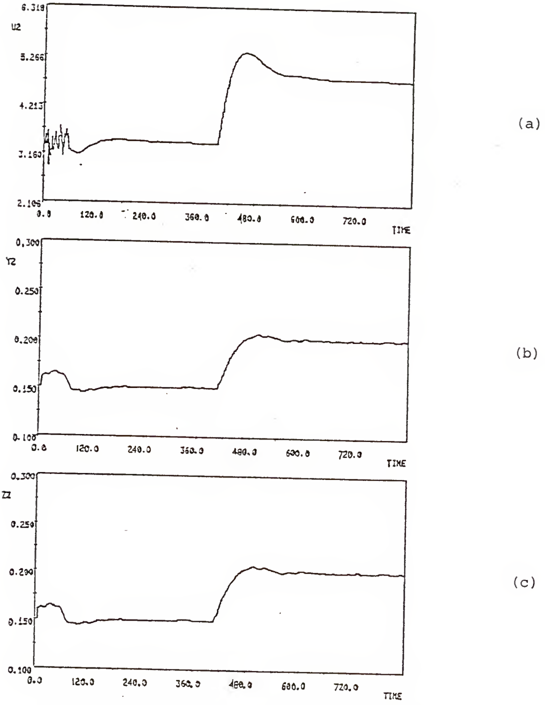

Figure 2-2. Response of the "standard" STC
(a) Flowrate (scaled) vs. time (min).
(b) Concentration measurement (scaled) vs. time (min).
(c) Actual concentration (scaled) vs. time (min).

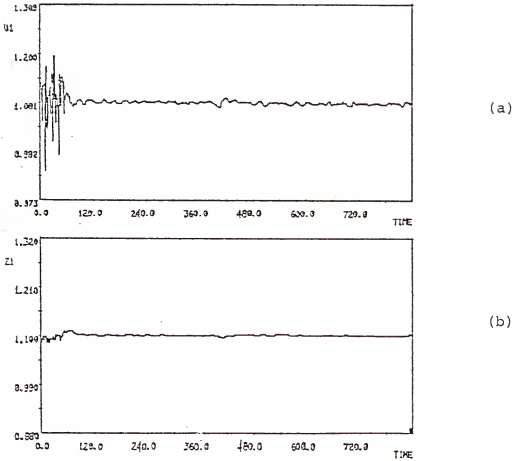

Figure 2-3. Response of the "simplified" STC
(a) Coolant temperature (scaled) vs. time (min).
(b) Actual reactor temperature (scaled) vs. time (min).

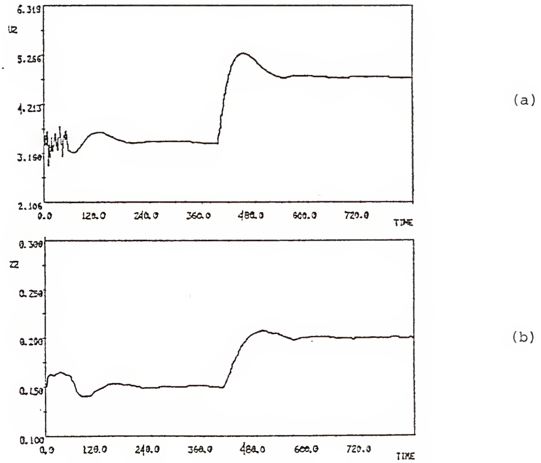

Figure 2-4. Response of the "simplified" STC
(a) Flowrate (scaled) vs. time (min).
(b) Actual concentration (scaled) vs. time (min).

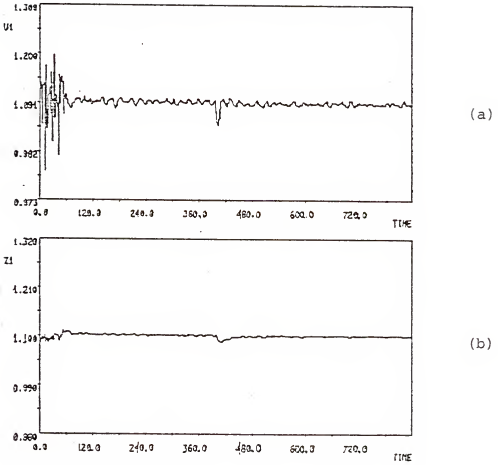

Figure 2-5. Response of the "decoupled" STC
(a) Coolant temperature (scaled) vs. time (min).
(b) Actual reactor temperature (scaled) vs. time (min).

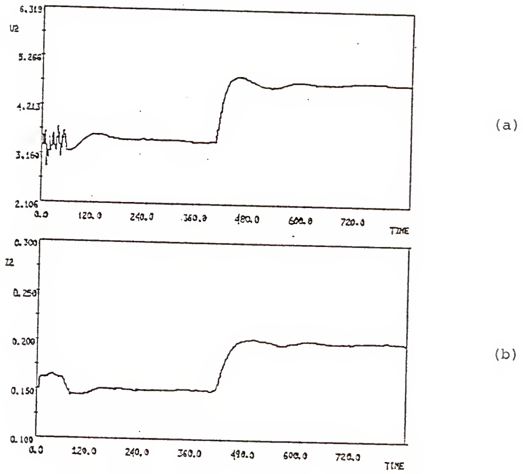

Figure 2-6. Response of the "decoupled" STC
(a) Flowrate (scaled) vs. time (min).
(b) Actual concentration (scaled) vs. time (min).

information for the parameter updating, the "simplified" algorithm simplifies the "standard" one and reduces the amount of on-line calculation. This usually brings a small deterioration of performance.

Adaptive Controllers for Multivariable Systems with Measurement and Control Delays

Self-tuning controllers are developed for the control of multiple-input multiple-output discrete systems with measurements and controls of individual time delays, i.e., systems in which each measurement and each control variable is subject to a distinct time delay. Two self-tuning algorithms are derived, one direct (i. e., it calculates the control policy directly using the parameters given from a least squares recursive parameter estimator), and one indirect (e.g., it does not estimate the control policy parameters but calculates them from model parameter estimates via recursive formulae). A simulation example demonstrates the attractive properties of these algorithms.

Model

The model of order n+1 has the form

$$A(q^{-1})z(t+1) = s+B(q^{-1}) \begin{bmatrix} u_1(t-r_1) \\ u_2(t-r_2) \\ \cdot \\ \cdot \\ \cdot \\ u_m(t-r_m) \end{bmatrix} \tag{2-16}$$

where r_i is the delay according to which the ith input affects the process. The rest of the notation is the same as in (2-1). Without any loss of generality, it can be assumed that

$$r_i \leqslant r_{i+1} \qquad i=1,2,\ldots m-1$$

Denoting

$$w(t)' = [y_1(t+k_1-k_\ell),\ y_2(t+k_2-k_\ell)\ldots y_\ell(t)]$$

and (2-17)

$$v(t)' = [u_1(t-r_1+r_m),\ u_2(t-r_2+r_m)\ldots u_m(t)]$$

system (2-16) can be rewritten as

$$A(q^{-1})w(t+d+1) = s+B(q^{-1})v(t) \qquad (2\text{-}18)$$

or

$$w(t+d+1) = s^*+C(q^{-1})w(t)+D(q^{-1})v(t) \qquad (2\text{-}19)$$

where $d = k_\ell + r_m$ and the rest of the notation is as in (2-4), (2-5), and (2-6) for $k = d$.

Equation (2-19) leads to the quadratic performance measure

$$J(v(t)) = e(t+d+1)'Q_1e(t+d+1)+\bar{v}(t)'Q_2v(t)+\tilde{v}(t)'Q_3\tilde{v}(t) \qquad (2\text{-}20)$$

with

$$e(t) = w(t)-y_s$$
$$\bar{v}(t) = v(t)-u_s$$
$$\tilde{v}(t) = v(t)-v(t-1)$$

Also Q_1, Q_2, and Q_3 are weight matrices. Minimization of $J(v(t))$ with respect to $v(t)$ and substitution of the true parameters by their estimates at time t gives

$$v(t) = -[D_t^{(0)}{}'Q_1D_t^{(0)}+Q_2+Q_3]^{-1}[D_t^{(0)}{}'Q_1c(t)-Q_2u_s-Q_3u(t-1)] \qquad (2\text{-}21)$$

where

$$c(t) = \hat{s}_t^* + \hat{C}_t(q^{-1})w(t) + \sum_{i=1}^{n+d} \hat{D}_t^{(i)} v(t-i)$$

It is noted that $(r_m - r_1)$ initial conditions on $v(t)$ are needed in order to determine $u(t)$ from $v(t)$ since

$$u_i(t) = v_i(t+r_i-r_m) \qquad i=1,2,\ldots,m$$

Parameter Estimation

Direct estimation of the controller parameters

Equation (2-19) is written in vector form

$$w(t) = \Theta x(t-1) \tag{2-22}$$

where

$$\Theta = [s^* C^{(0)} C^{(1)} \ldots C^{(n)} D^{(0)} \ldots D^{(n+d)}]$$

$$x'(t-1) = [1\ w'(t-d-1) \ldots w'(t-d-n-1) v'(t-d-1) \ldots v'(t-2d-n-1)]$$

The rows of θ can then be estimated by any of the sequential algorithms of Appendix A.

Indirect estimation of the controller parameters

Equation (2-16) is written as

$$w(t) = \Theta x(t-1)$$

where

$$\Theta = [s\ A^{(0)} \ldots A^{(n)} B^{(0)} \ldots B^{(n)}]$$

$$x(t-1) = [1\ w'(t-1) \ldots w'(t-n) v'(t-d-1) \ldots v'(t-d-n-1)]$$

The parameters in θ are estimated as before. After $\Theta(t)$ is obtained at each sampling period, the controller parameters in (2-21) are calculated from equation (2-6) for $k = k_\ell + r_m$; a proof of (2-6) is given in Appendix C.

The estimation scheme of both the direct and the indirect schemes involves only one gain vector, one covariance matrix of dimensions $[1+\ell(n+1)+m(n+1)] \times [1+\ell(n+1)+m(n+1)]$, and one regression vector $x(t-1)$.

The direct algorithm has the advantage over the indirect that the computation of the control policy is considerably simpler. However, there is a trade off, namely that more parameters need to be estimated. Also, the direct scheme uses information obtained at time $t-k_\ell-r_m-1$ and previously, while the indirect scheme uses information obtained at $t-1$ (more recent). Another advantage of the indirect scheme is that it requires less initialization time than the direct one.

Example

The following first order open loop unstable discrete system is considered:

$$\begin{bmatrix} z_1(t+1) \\ z_2(t+1) \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} -1.0 & 0.5 \\ 0.7 & -0.2 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \begin{bmatrix} 1.0 & -0.3 \\ 1.5 & 1.0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t-1) \end{bmatrix}$$

$$y_1(t) = z_1(t)$$

$$y_2(t) = z_2(t-2)$$

Here $n = 0$, $\ell = m = 2$, $k_1 = 0$, $k_2 = 2$, $r_1 = 0$, and $r_2 = 1$. The vector s was held at $(0,0)'$ until the sampling time $t = 200$, when it was changed to $(1,1)'$. The algorithms were initialized at the same initialization time t_i. The estimation algorithm RLS-VFF2 from Appendix A was used. The off-line choices were:

$$Q_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

and

$N_1 = 1.0$ $\lambda_{min} = 0.1$ for the indirect

$N_1 = 1.0$ $\lambda_{min} = 0.7$ for the direct

(The larger number of parameters in the direct estimation required a higher bound to the maximum forgetting factor λ).

The performance of the algorithms is shown in Figures (2-7) and (2-8). It is seen that both succeed quite well in adjusting the control policy to the new optimal value after the load change at time $t = 200$ sampling interval.

Conclusions

Multivariable systems in which each measurement and each control variable has an individual time delay are frequently encountered in practice. The self-tuning controllers developed in this section successfully control such systems, while being conceptually simple. The indirect algorithm uses the most recent information available for the parameter estimation and requires less initialization time. The direct algorithm estimates a larger number of parameters but involves less computation to obtain the control policy. In the simulation example, both algorithms successfully handled a large load change. Their performances were comparable.

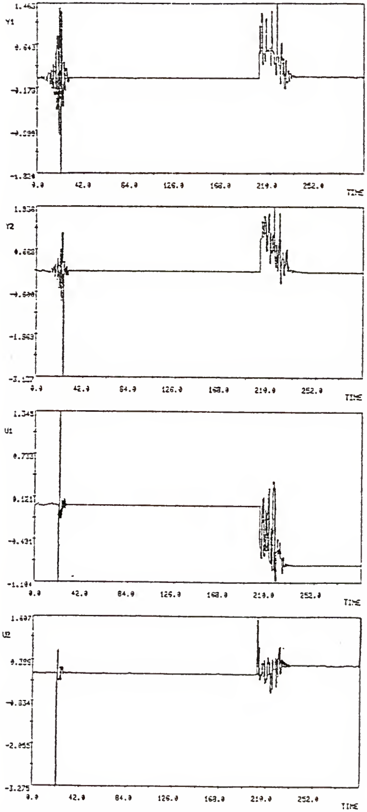

Figure 2-7. Performance of the indirect STC.

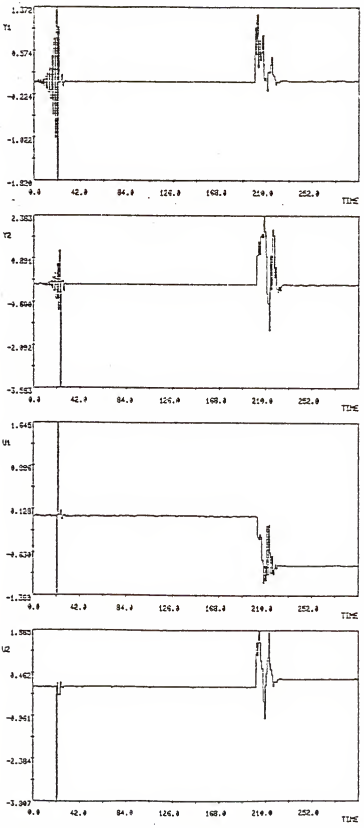

Figure 2-8. Performance of the direct STC.

CHAPTER III
MULTIVARIABLE ADAPTIVE AND INTERNAL MODEL CONTROLLER DESIGN VIA FACTORIZATION METHODS

3.1 Introduction

The design of single-input single-output (SISO) adaptive controllers in which the control law is desired to be linear with respect to the estimated parameters requires knowledge of the scalar nonzero leading input coefficient $b^{(0)}$. In the STC of Clarke and Gawthrop (21), one may only need to know á priori the sign of $b^{(0)}$ but then the off-line weight choices do not correspond exactly to the performance measure in the adaptive scheme. The situation is even more complicated in the multivariable case where the matrix $B^{(0)}$, the counterpart of $b^{(0)}$, should be nonsingular (see (4-11), (4-12)) and known á priori. Furthermore, the matrix $B^{(0)}$ is obtained after extracting a delay part from the matrix $B(q^{-1})$ (otherwise $B^{(0)} = B(0)$ is in general singular) and the invertibility properties of $B^{(0)}$ are therefore dependent on the way this delay extracting is done. The delay extraction corresponds in general to a factorization of the process transfer function and in particular of the polynomial matrix $B(q^{-1})$. To be more specific in stating the previous results, we assume the plant to be linear, discrete-time, and modeled by the state equations

$$\begin{aligned} x(t+1) &= Fx(t)+Gu(t) \\ y(t) &= Hx(t) \end{aligned} \qquad (3\text{-}1)$$

where u(t) and y(t) are the ℓ-dimensional input and output deviation vectors, respectively, and x(t) is the n-dimensional state vector. The results of this section can also be applied to systems in which the number of inputs exceeds that of outputs. System (3-1) corresponds to an input-output description of the form

$$A(z)y(z) = B(z)u(z) \tag{3-2}$$

where A(z) and B(z) are polynomial matrices of appropriate dimensions in z and y(z) and u(z) denote the z-transforms of y(t) and u(t), respectively. B(z) in general has ijth entry equal to

$$z^{-d_{ij}}b_{ij}(z) \tag{3-3}$$

where d_{ij} denotes the time delay between the jth input and the ith output (for a strictly proper system $d_{ij} \geqslant 1$) and for $i,j = 1,2,\ldots,\ell$

$$b_{ij}(z) = b_{ij}^0+b_{ij}^1z^{-1}+\ldots+b_{ij}^{\beta ij}z^{-\beta ij}; \qquad b_{ij}^0 \neq 0$$

Also the ijth entry of A(z) can be assumed to be

$$a_{ij}(z) = 1+a_{ij}^1z^{-1}+a_{ij}^2z^{-2}+\ldots+a_{ij}^{\alpha ij}z^{-\alpha ij}$$

Form (3-2) corresponds to a left matrix fraction description (22) of (3-1), i.e.,

$$y(z) = G(z)u(z)$$
$$G(z) = A(z)^{-1}B(z)$$

where G(z) is the system transfer function matrix, which is assumed to be strictly proper, i.e., it satisfies

$$\lim_{z \to \infty} G(z) = 0$$

and of rank equal (23) to ℓ, the number of outputs. In the case of more inputs being available than outputs, one can consider only the inputs which correspond to any ℓ-set of linearly independent columns of G(z). This makes output controllability possible (24).

In Borisson (14) and Koivo (17), it is assumed that all the delays d_{ij} are equal to k+1. In Goodwin et al. (15) as well as in Tsiligiannis and Svoronos (24), a different delay was associated with each output. The notion of the interactor matrix introduced by Wolowich and Falb (8) was used in Goodwin and Long (25) and in Elliot and Wolowich (26) to extract the delay part of the transfer function in order to obtain a system parametrization with a nonsingular $B^{(0)}$. It turns out that knowledge of the interactor matrix is equivalent to knowing the Hermite canonical form of the process transfer function, a requirement which is not in general met by systems which are to be adaptively controlled. The problem of obtaining a parametrization of the system with nonsingular $B^{(0)}$ was also addressed in Johansson (27) and (28), where the developed parametrizations were based on the structure matrix concept introduced by Pernebo (29). However, á priori information is needed by this approach also, since knowledge of the structure matrix is essentially knowledge of the Smith-McMillan canonical form of the system transfer function.

In what follows, first a delay factorization procedure is given which leads to a parametrization of the model with nonsingular $B^{(0)}$ and is therefore suitable for STC design. The method does not require knowledge of b_{ij} parameters and is applicable in general to two-dimensional systems and to a larger class of higher dimensional systems than those treated in Goodwin et al. (15) or in Tsiligiannis and Svoronos (24). The latter correspond to the case of a diagonal interactor matrix. Second, the notion of system invariants is discussed and the right interactor is introduced in addition to the existing (left) interactor. The right interactor is shown to be a process invariant for disturbance estimation in the same way as the left interactor is an invariant under feedback control. Next the left interactor is used for designing the best possible IMC controller in case of delays and no finite unstable zeros present in the system. In a dual way, the right interactor is used for designing the perfect disturbance estimator. This is followed by an adaptive control algorithm design based on the right interactor. Finally, the notions of generalized left and right interactor matrices are introduced and used for IMC controller and feedforward estimator design for processes with multiple delays, multiple unstable zeros, and non-trivial cross-couplings.

3.2 Multivariable STC Design Via Delay Factorization

A procedure which leads to a factorization of the process delays in such a way that the resulting model has a nonsingular $B^{(0)}$ is given in this section. Due to the facts that the polynomial matrix $A(q^{-1})$ can always be diagonalized (see (2-9)) and that the process delays are greater than or equal to one, we can assume the model to be of the form

$$\begin{aligned} A(q^{-1})y(t+1) &= s+B'(q^{-1})u(t) \\ A(q^{-1}) &= \mathrm{diag}\,[a_i(q^{-1})] \end{aligned} \qquad (3\text{-}4)$$

with

$$a_i(q^{-1}) = 1+a_{i1}q^{-1}+a_{i2}q^{-2}+\dots+a_{ia_i}q^{-a_i}$$

and

$$[B'(q^{-1})]_{ij} = q^{-d_{ij}}b_{ij}(q^{-1}) \qquad d_{ij} \geqslant 0$$

with

$$b_{ij}(q^{-1}) = b_{ij0}+b_{ij1}q^{-1}+\dots+b_{ijb_{ij}}q^{-b_{ij}}; \; b_{ij0} \neq 0 \text{ for all } i,j$$

Thus, d_{ij} is the time delay between control u_j and measurement y_i, in addition to the inherent delay of one sampling interval present in discrete time systems. Let

$$\det B'(q^{-1}) = b_0q^{-p}+b_1q^{-p-1}+\dots+b_{p'}q^{-p-p'}; \; b_0 \neq 0$$

Also let d^*_{ij1}, $i = 1,2,\dots,\ell$ be a set of delays which form the leading exponent of det B (q^{-1}), i.e.,

$$d^*_{1j1}+d^*_{2j2}+\dots+d^*_{\ell j\ell} = p.$$

It is assumed that these d^*_{iji} (and hence p) are known. The rest of the delays need not be known but knowledge of a lower bound $\underline{d}_{ij}$ is required for every delay d_{ij}. In addition, it is assumed that upper bounds to the orders of the polynomial matrices are known.

Consider now the following optimization problem: find nonnegative integers k_i and r_i, $i = 1,2,\ldots,\ell$ which maximize

$$I = \sum_{i=1}^{\ell} k_i+r_i \,, \tag{3-5}$$

subject to

$$k_i+r_j \leqslant \underline{d}_{ij}, \text{ for } i,j = 1,2,\ldots,\ell.$$

This is a standard linear integer programming problem and can be easily solved, for example, by the methods given in Kaufmann and Laborděre (30). Then the matrices

$$K(q) = \operatorname{diag}(q^{k_i}) \,,$$
$$R(q^{-1}) = \operatorname{diag}(q^{-r_i})$$

are formed and then model (3-4) is equivalent to the model

$$A(q^{-1})K(q)y(t+1) = s+B(q^{-1})R(q^{-1})u(t) \tag{3-6}$$

with

$$A(q^{-1}) = \sum_{i=0}^{n} A^{(i)}q^{-i-1} \,,$$
$$B(q^{-1}) = K(q)B'(q^{-1})R(q) = \sum_{i=0}^{n} B^{(i)}q^{-i}$$

where $A^{(i)}$ and $B^{(i)}$ are constant $\ell x \ell$ matrices and

$$n \geq \max\{\max_i(a_i-1),\ \max_{ij}(d_{ij}+b_{ij}-k_i-r_j)\}.$$

Note that model (3-6) is model (2-18) with

$$w(t+k) = K(q)y(t);\ k = \max_i\{k_i\},$$

$$v(t) = R(q^{-1})u(t+r);\ r = \max_i\{r_i\}.$$

It should be pointed out that lumped parameter systems result naturally in form (3-6), in which there is a delay k_i associated with each measurement y_i and a delay r_i associated with each control u_i. However, experimental determination of the time delays leads to form (3-4).

The following theorem establishes the equivalence of models (3-4) and (3-6).

Theorem 3.1. Let $B'(q^{-1})$ be as in (3-4) with

$$\det(B'(q^{-1}) = b_0 q^{-p} + \text{h.o.t.},\ b_0 \neq 0 \text{ (h.o.t. denotes higher order terms in } q^{-1})$$

Then

(i) for any set of k_i's and r_i's which solve the optimization problem (3-5) the factorization

$$B'(q^{-1}) = K(q^{-1})\ B\ R(q^{-1})$$

with

$$K(q^{-1}) = \mathrm{diag}(q^{-k_i})$$

and

$$R(q^{-1}) = \mathrm{diag}(q^{-r_i})$$

has

$$B(0) \quad (=B^{(0)} = D^{(0)})$$

nonsingular if and only if

$$I_{max} = p.$$

(ii) If $\ell = 2$ (two control and measured variables), all solutions of (3-5) result in a nonsingular $B(0)$ provided

$$p = \min(d_{11}+d_{22}, d_{12}+d_{21}).$$

Proof:

(i) Given any set k_i, r_i such that

$$k_i+r_i \leqslant d_{ij}, \text{ all } i,j$$

the matrix

$$B = K(q)B'(q^{-1})R(q) \tag{3-7}$$

does not include a power of q higher than zero and therefore can be written as

$$B(q^{-1}) = \sum_{i=0}^{n} B^{(i)}q^{-i}.$$

From

$$B'(q^{-1}) = K(q^{-1})B(q^{-1})R(q^{-1})$$

taking determinants, it follows that

$$b_0q^{-p}+\text{h.o.t.} = q^{-\sum_{i=1}^{\ell} k_i+r_i}[\det B(0)+\text{h.o.t.}]. \tag{3-8}$$

From (3-8) it follows that

$$p \geqslant \sum_{i=1}^{\ell} k_i+r_i$$

and that

$$\det (B(0)) \neq 0$$

if and only if

$$\sum_{i=1}^{\ell} k_i+r_i = p .$$

(ii) This part is proved directly by construction. The inputs and outputs can be rearranged so that

$$d_{11}+d_{22} \leqslant d_{12}+d_{21}$$

and

$$d_{12} \geqslant d_{21}.$$

Then if (a)

$$d_{21} \geqslant d_{11}+d_{22},$$

a solution is

$$k_1 = 0,\ k_2 = d_{22},\ r_1 = d_{11},\ r_2 = 0$$

and if (b)

$$d_{21} < d_{11}+d_{22} ,$$

a solution in case (b_1)

$$d_{21} \geqslant d_{22} ,$$

is

$$k_1 = d_{11}+d_{22}-d_{21},\ k_2 = d_{22},\ r_1 = d_{21}-d_{22},\ r_2 = 0$$

and a solution in case (b_2)

$$d_{21} < d_{22} ,$$

is

$$k_1 = d_{11},\ k_2 = d_{21},\ r_1 = 0,\ r_2 = d_{22}-d_{21}.$$

Since all these solutions give

$$\sum_{i=1}^{2} k_i+r_i = d_{11}+d_{22}$$

and

$$k_i+r_j \leqslant d_{ij},$$

it follows from the assumption on p, which is generically true, that B(0) is nonsingular.

Remark: From Theorem 3.1, it follows that all systems satisfying the condition of Lemma 3.2 for $z = 0$ of Goodwin et al. (15), i.e., systems with diagonal interactor matrix, can be written in the form (3-6) with generically nonsingular $B^{(0)}$ and with

$$k_i = \min_j d_{ij}, \ r_i = 0, \qquad i = 1,2,\ldots\ell.$$

Theorem 3.1 states that the matrix $B^{(0)}$ is nonsingular if and only if $I_{max} = p$, a condition easily checked once the k_i and r_i have been determined. This condition will be generally true for $\ell = 2$, while for $\ell > 2$ it will be true for a larger class of systems than those treated in Goodwin et al. (15). It could also happen that the integer programming problem (3-5) has multiple solutions. In this case, the solution with minimum k_j, where y_j is the most important output, is chosen. Finally, it is noted that the calculation of k_i and r_i is done off-line and involves no unknown parameters.

Examples: This example treats a three-dimensional system with an unknown and possibly nondiagonal interactor matrix. Let the system be

$$y(t+1) = \begin{bmatrix} b_{11}q^{-3} & b_{12}q^{-5} & b_{13}q^{-5} \\ b_{21}q^{-d21} & b_{22}q^{-2} & b_{23} \\ b_{31}q^{-d31} & b_{32}q^{-1} & b_{33}q^{-d33} \end{bmatrix} u(t) \qquad b_{ij} \neq 0 \tag{3-9}$$

The unspecified delays (d_{21}, d_{31}, d_{33}) are unknown nonnegative integers. The information available allows us to set $p = 4$. Solution of the optimization problem (3-5) using the lower bound zero for the unspecified delays gives

$$k_1 = 3,\ r_2 = 1$$

and

$$k_2 = k_3 = r_1 = r_3 = 0.$$

Thus, (3-9) can be rewritten in the form of (3-6) as

$$w(t+1) = \sum_{i=1}^{n} B^{(i)} v(t-4-i)$$

with

$$B^{(0)} = \begin{bmatrix} b_{11} & 0 & 0 \\ x & 0 & b_{23} \\ x & b_{32} & x \end{bmatrix}$$

where x denotes an element which may or may not be zero. Clearly, since

$$b_{11},\ b_{23},\ b_{32} \neq 0,$$

$$\det B^{(0)} \neq 0.$$

Note that if d_{31} or d_{33} equal zero, the interactor matrix is not diagonal. This system can be treated by the STC presented here even though the interactor is not fully known.

The following example constitutes a case where the algorithm will not result in a nonsingular $B^{(0)}$. Let

$$B'(q^{-1}) = \begin{bmatrix} 1 & q^{-1} & q^{-1} \\ 1 & 2q^{-1} & q^{-1} \\ 1 & 1 & 1 \end{bmatrix}$$

Then p = 1 but since

$$d_{i1} = 0,\ d_{3j} = 0 \qquad i,j = 1,2,3$$

we must have

$$k_i = r_i = 0 \qquad i = 1,2,3.$$

Therefore, $\sum_{i=1}^{3} k_i+r_i = 0 < 1$

and $B^{(0)}$ will be singular. Indeed,

$$B^{(0)} = B'(0) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

which is singular.

The next example shows that the solution to (3-5) is not necessarily obtained by first extracting the k_i's as the minimum delays of each row and then extracting the r_i's. Let

$$B'(q^{-1}) = \begin{bmatrix} q^{-1} & q^{-5} & q^{-6} \\ q^{-1} & q^{-4} & q^{-10} \\ q^{-3} & q^{-3} & q^{-4} \end{bmatrix}$$

Here $p = 9$. Taking

$$k_1 = 1,\ k_2 = 1,\ k_3 = 3,$$

we see that

$$r_1 = 0,\ r_2 = 0,\ \text{and } r_3 \leqslant 1$$

But then

$$p = 9 > 6 \geqslant \sum_{i=1}^{3} k_i + r_i$$

and the resulting $B^{(0)}$ is singular. The same example shows that extraction of the r_i's as the minimum delays in each column and then extraction of k_i's will also result in a singular $B^{(0)}$. For, taking

$$r_1 = 1,\ r_2 = 3,\ r_3 = 4,$$

we obtain

$$k_1 = k_2 = k_3 = 0$$

and

$$p = 9 > 8 = \sum_{i=1}^{3} k_i + r_i .$$

However, the solution of (3-5),

$$k_1 = 1,\ k_2 = 1,\ k_3 = 0,\ r_1 = 0,\ r_2 = 3,\ r_3 = 4$$

results in a nonsingular $B^{(0)}$.

The following key observation will allow us to design an implementable control law for the model (3-6).

Lemma 3.2. If

$$R(q^{-1}) = \text{diag } q^{-r_i} \qquad i = 1,2,\ldots,\ell$$

and

$$v(t) = q^r R(q^{-1})u(t)$$

where

$$r = \max_i\{r_i\},$$

then u(t) can be determined if

$$v(t),\ v(t-1),\ldots v(t-r+m)$$

are known, where

$$m = \min_i\{r_i\}.$$

In other words, u(t) can be determined at every t after an initialization period of r-m+1 steps.

Proof: We have

$$u(t) = q^{-r} R(q)v(t)$$

which includes no predictive elements since it is a polynomial matrix in q^{-1}. Moreover, if

$$r = r_j$$

then

$$u_j(t) = v_j(t)$$

and all other components of $u(t)$ correspond to

$$v(t-1),\ v(t-2), \ldots, v(t-r+m).$$

In particular, if

$$m = r_m,$$

then

$$u_m(t) = v_m(t-r+m).$$

Note that $q^r\ R(q^{-1})$ here being stably invertible guarantees that it will have a causal inverse since it is diagonal. In general, however, stable invertibility does not guarantee causality of the inverse. This is shown by

$$T(z) = \begin{bmatrix} z & 0 \\ z^3 & z \end{bmatrix}$$

which is stably invertible but which has no causal inverse since

$$T(z)^{-1} = \begin{bmatrix} z^{-1} & 0 \\ -z & z^{-1} \end{bmatrix}.$$

Based now on model (3-6), we can design the control law which will give $v(t)$ at each sampling instant. Denoting by

y_s and u_s the desired steady state values, (3-6) suggests the following performance measure in parallel to (2-16), (2-17), (2-18), (2-19), and (2-20):

$$J(v(t)) = \| P(q^{-1})[w(t+d+1)-y_s] \|^2 + \| Q'(q^{-1})[v(t)-u_s] \|^2 \tag{3-10}$$

where

$$w(t) = q^k K(q)y(t), \; k = \max_i k_i, \text{ and } d = k+r$$

and where $P(q^{-1})$, $Q'(q^{-1})$ are $\ell x \ell$ matrix polynomials in q^{-1} and $P(0)$ is assumed to be nonsingular. In industrial processes, u_s is frequently known from steady state operation data. In case it is unknown, one chooses $Q'(q^{-1})$ such that $Q'(1) = 0$.

Assuming that $B^{(0)}$ is nonsingular, $(\sum_{i=1}^{\ell} k_i + r_i = p)$, the optimum controller sets:

$$P(q^{-1})w(t+d+1)+Q(q^{-1})[v(t)-u_s] = P(1)y_s \tag{3-11}$$

where

$$Q(q^{-1}) = (P^T(0))^{-1}(B^{(0)T})^{-1}Q'^T(0)Q'(q^{-1}).$$

The off-line choice for the input weight is now $Q(q^{-1})$ instead of $Q'(q^{-1})$. Guidelines for choosing it are given in Koivo (17).

The self-tuning control law is:

$$P(q^{-1})[\hat{s}(t)+\sum_{i=0}^{n} \hat{C}^{(i)}(t)w(t-i)+\sum_{i=0}^{n+d} \hat{D}^{(i)}(t)v(t-i)]+ Q(q^{-1})[v(t)-u_s] = P(1)y_s \tag{3-12}$$

where $\hat{\cdot}(t)$ denotes estimate of $\cdot$ at time t. An analysis and proof of asymptotic stability for this algorithm is given in Chapter IV.

3.3 Design of Multivariable Adaptive and Internal Model Controllers via System Invariants

As the theory for single-input, single-output adaptive and internal model control has solidified in the last decade, attention has turned toward extension of the main principles to multiple-input, multiple-output systems. Issues such as stability of multivariable systems under a certain feedback control action, decoupling, multivariable time delay compensation, and disturbance estimation in multivariable processes are intimately related to the structure of the process transfer matrix. In particular, there are some process characteristics which include time delays and inverse response behavior which cannot be affected by any stable linear feedback controller and which will therefore appear necessarily in the system's response. The optimal feedback controller allows for a minimal effect of such characteristics on the output response. Apart from control, it can be used for evaluation of design alternatives according to their operability features. The best possible linear feedback controller is determined here by use of the left interactor and left generalized interactor matrices. Dual results are shown for disturbance estimation where the best possible feedforward estimator is found by use of the right interactors.

The process characteristics which make it impossible to have perfect control, i.e., all plant outputs equal to the desired set points for all times, via stable controller are pointed out next. Also the same characteristics are shown to forbid perfect disturbance estimation via a stable feedforward estimator. If a left coprime factorization of C and a right coprime factorization of G in Figure 3-1 are $C = Q^{-1}P$ and $G = BA^{-1}$, respectively, then

$$Qu = -Py+Py_s$$

and the closed loop is described by

$$y = B(PB+QA)^{-1}Py_s+B(PB+QA)^{-1}QAB^{-1}d.$$

Hence, any attempt to cancel the unstable zeros of B will introduce an unobservable unstable mode in the closed loop transfer function. Consequently, under any stable controller, the system will respond to set point changes according to the dead times and the inverse response characteristics of B. Consider now the problem corresponding to Figure 3-2 where we want an estimate $\hat{d}$ of the unmeasurable disturbance d which affects the system G. If $G = BA^{-1}$ and $E = PQ^{-1}$ are right coprime factorizations of G and E, then

$$\hat{d} = PQ^{-1}BA^{-1}d$$

and no stable and proper estimator E can cancel the time delays and inverse response characteristics of B.

Next the above characteristics are extracted from the process transfer function by use of the interactor matrices. We start with the interactors which characterize the delay structure of the process.

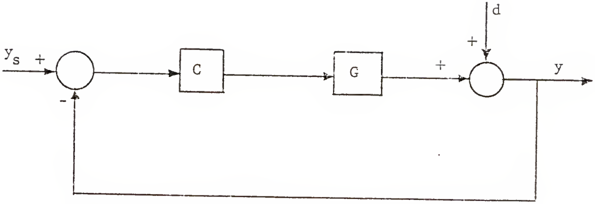

Figure 3-1. The classical feedback control loop.
C = controller, G = plant,
y_s = set-point, y = output,
d = disturbance.

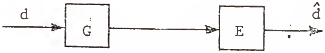

Figure 3-2. Feedforward disturbance estimator (E).

Lemma 3.3. For any proper $\ell x \ell$ transfer matrix $G(z)$ of rank ℓ, there exist two unique polynomial matrices, a lower triangular one, $\xi_G^\ell(z)$, and an upper triangular one, $\xi_G^r(z)$, called the left and right interactor matrices, respectively, which are of the form

$$\xi_G^\ell(z) = H_G^\ell(z)\,\mathrm{diag}[z^{k_1},\dots,z^{k_\ell}]$$

$$\xi_G^r(z) = \mathrm{diag}[z^{r_1},\dots,z^{r_\ell}]\,H_G^r(z)$$

$$H_G^\ell(0) = H_G^r(0) = I \qquad (H_G^\ell(z)_{ii} = (H_G(z))_{ii} = 1$$

$H_G^\ell(z)$ is a lower triangular polynomial matrix, $H_G(z)$ is an upper triangular polynomial matrix

and which have the following properties:

(i) $\lim_{z\to\infty} \xi_G^\ell(z)\ G(z) = K_G^\ell$ with K_G^ℓ nonsingular;

$\lim_{z\to\infty} G(z)\ \xi_G^r(z) = K_G^r$ with K_G^r nonsingular;

(ii) Both $\xi_G^\ell(z)$ and $\xi_G^r(z)$ are stably invertible;

(iii) Both $\xi_G^\ell(z)$ and $\xi_G^r(z)$ are causally invertible.

Proof:

(i) The left interactor was introduced in Wolowich and Falb (8). Let ′ denote transpose. It follows that for $G'(z)$ there exists $\xi_{G'}^\ell(z)$ such that

$$\lim_{z\to\infty} \xi_{G'}^\ell(z)\ G'(z) = K_{G'}^\ell,\ K_{G'}^\ell \text{ nonsingular}$$

or

$$\lim_{z\to\infty} G(z)\ \xi_{G'}^{\ell'}(z) = K_{G'}^{\ell'},\ K_{G'}^{\ell'} \text{ nonsingular.}$$

Thus

$$\xi_G^r(z) = \xi_{G'}^{\ell'}(z)$$

(ii) $\det \xi_G^r(z) = z^{\sum_{i=1}^{\ell} k_i} \neq 0$ for $|z| \geq 1$,

$\det \xi_G^\ell(z) = z^{\sum_{i=1}^{\ell} r_i} \neq 0$ for $|z| \geq 1$, QED.

(iii) Let $\xi_G^\ell(z)\, G(z) = K_G^\ell(z)$. Then

$$\lim_{z\to\infty} \xi_G^\ell(z)^{-1} = \lim_{z\to\infty} G(z)\, K_G^\ell(z)^{-1} = G(\infty)\, K_G^{\ell-1}$$

which is finite since G is strictly proper and K_G^ℓ is nonsingular. Then $\xi_G^{\ell-1}$ is strictly proper, or ξ_G^ℓ is causally invertible.

Lemma 3.4. Let G(z) be as in Lemma 3.3. Then $G(z)^{-1}\, \xi_G^\ell(z)^{-1}$ and $\xi_G^r(z)^{-1}\, G(z)^{-1}$ are proper.

Proof: We have that

$$\lim_{z\to\infty} G(z)^{-1}\, \xi_G^\ell(z)^{-1} = K_G^{\ell-1}$$

and

$$\lim_{z\to\infty} \xi_G^r(z)^{-1}\, G(z)^{-1} = K_G^{r-1}$$

and both $K_G^{\ell-1}$ and K_G^{r-1} are finite since K_G^ℓ and K_G^r are nonsingular.

Theorem 3.5. Let $G_1(z)$, $G_2(z)$ be as in Lemma 3.1. Then there exists a proper G(z) such that

(i) $G_1(z)\, G(z) = G_2(z)$ if and only if $\xi_{G_1}^\ell(z)\, \xi_{G_2}^\ell(z)^{-1}$ is proper

(ii) $G(z)\, G_1(z) = G_2(z)$ if and only if $\xi_{G_2}^r(z)^{-1}\, \xi_{G_1}^r(z)$ is proper.

Proof: The proof for (ii) given here is analogous to the one for (i) given in (8).

$$G(z) = G_2(z)\ G_1^{-1}(z) =$$

$$G_2\ \xi^{r}_{G_2}(z)\ (\xi^{r}_{G_2}\ (z)^{-1}\ \xi^{r}_{G_1}(z))\ (\xi^{r}_{G_1}(z)^{-1}\ G_1(z)^{-1})$$

which by Lemma 3.4 shows (ii).

Application of Theorem 3.5 for $G_2(z) = I$ gives

Corollary 3.6. If $G_1(z)$ is proper, then

(i) $G_1(z)$ has a proper right inverse if and only if $\xi^{\ell}_{G_1}(z) = I$

(ii) $G_1(z)$ has a proper left inverse if and only if $\xi^{r}_{G_1}(z) = I$.

The proof is a direct consequence of the fact that

$$\xi^{r}_{I}(z) = \xi^{\ell}_{I}(z) = I.$$

Internal Model Controller Design via the Left Interactor

The above results are used here in designing internal model controllers for systems with multiple delays. The argument (z) and the ~ are dropped in what follows for notational simplicity.

The standard feedback controller structure of Figure 3-1 which corresponds to

$$y = G(I+CG)^{-1}C(y_s-d)+d$$

$$u = (I+CG)^{-1}C(y_s-d)$$

is equivalent (31) to the internal model control structure shown in Figure 3-3, with

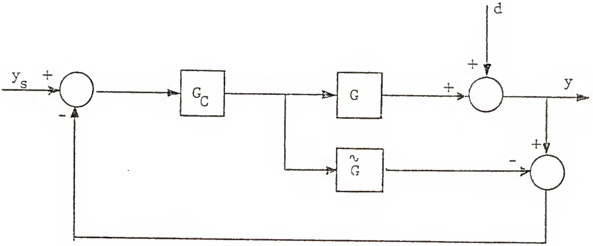

Figure 3-3. The internal model control configuration.

$$G_C = C(I+\tilde{G}C)^{-1}$$

$$C = G_C(I-\tilde{G}G_C)^{-1}$$

where $\tilde{G}$ is an assumed plant model. The closed loop behavior is described then by

$$y = GG_C(I+(G-\tilde{G})G_C)^{-1}(y_s-d)+d$$

from which if $\tilde{G}$ is stably invertible, i.e., if det $\tilde{G}(z) \neq 0$ for all complex z with $|z| \geq 1$, then the choice $G_C = \tilde{G}^{-1}$ will yield (6) the best possible output response. However, due to the presence of unstable zeros including the ones at infinity which are due to the presence of time delays, $\tilde{G}$ is never stably invertible, for discrete-time systems. This is justified since $\tilde{G}(z)$ is strictly proper, due to the inherent time delay of one sampling interval. Then it can be seen by the construction of $\xi^{\ell}_{\tilde{G}}$ in Appendix B that $k_i > 0$, $i = 1,2,\ldots,\ell$. Consequently, $\xi^{\ell}_{\tilde{G}}$ is never equal to I and by corollary 3.6, $\tilde{G}(z)$ does not have a proper right inverse.

In the following, the necessary stability condition for internal model controller design, i.e., that the system is open loop stable, is made. Also it is assumed that $\tilde{G}$ has no finite unstable zeros, an assumption that will be relaxed later.

The delay structure of the process is taken care of in the IM controller design by:

Theorem 3.7. (Characterization of all admissible controllers). Assume that a linear system with transfer matrix G(z) is open loop stable and has all finite transmission zeros inside the unit circle. Then for any proper and stable (no poles on or outside the unit circle) transfer matrix T with $T(1) = I$, the IM controller

$$G_C(z) = G(z)^{-1}\ \xi_G^{\ell}(z)^{-1}\ \xi_G^{\ell}(1)T(z)$$

is proper and stable and eliminates steady state offset. It is noted that T will in general contain the IM filter which enhances the robustness of the closed loop.

Proof: From Lemma 3.4, G_C is proper. It will be shown that G_C is also stable. Since ξ_G^{ℓ} is stably invertible (Lemma 3.3 (ii)), G has no finite unstable zeros, and T is stable, G_C is also stable. Finally, since the closed loop transfer function at $z = 1$ is equal to I, $G_C(z)$ eliminates steady state offset.

Theorem 3.8. (Design of optimal IM controllers). Assume that the plant $G(z)$ has no finite zeros on or outside the unit circle.

(i) For step changes in the set-point vector, the IM controller

$$G_C(z) = G(z)^{-1}\ \xi_G^{\ell}(z)^{-1}\ \xi_G^{\ell}(1) \tag{3-13}$$

minimizes $t_1, t_2, \ldots, t_\ell$ where t_1 is the time so that for $t \geqslant t$, $y_1(t) = y_{s1}$, and in general t_i is the time so that for $t \geqslant t_i$, $y_i(t) = y_{si}$, provided that $t_1, t_2, \ldots, t_{\ell-1}$ are minimized. In other words, output y_i reaches its set-point as soon as possible without adversely affecting a lower indexed output.

(ii) If $(\xi_G^{\ell}(z))_{ii} = z^{k_i}$, then

$$t_i = t_* + k_i; \quad i = 1,2,\ldots,\ell \tag{3-14}$$

where t_* is the time at which the set-point change occurred.

(iii) If ξ_G^{ℓ} is diagonal, the IM controller (3-13) is optimal with respect to any sum squared error (SSE) or sum absolute error (SAE) criterion for step changes in the set-point vector. Furthermore, it provides complete decoupling.

(iv) The IM controller (3-13) minimizes the SSE:

$$\sum_{t=t^*}^{\infty} e'(t) \lim_{w\to\infty} \begin{bmatrix} w^{\ell-1} & 0 & \cdots & 0 & 0 \\ 0 & w^{\ell-2} & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & w & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} e(t)$$

with $e = y_s - y$

or the analogous SAE.

Proof:

(i) Clearly, the performance index dictates that if y_i is at its set-point, it must not be moved by a set-point change in y_j for $j > i$. This directly implies that the optimal closed loop transfer matrix $G_{CL}(z)$ must be lower triangular. Now let $t_* + r_i; i = 1,\ldots,\ell$ be the minimal t_i. The definition of t_i and the above analysis imply that $G_{CL}(z)$ must have the form

$$M(z) = (G_{CL}(z))^{-1} = L(z)\ \mathrm{diag}(z^{m_{ii}} \frac{a_{ii}(z^{-1})}{b_{ii}(z^{-1})}) \qquad (3\text{-}15)$$

where L(z) is a lower triangular matrix with diagonal elements 1 and $a_{ii}(z^{-1})$, $b_{ii}(z^{-1})$ are polynomials in z^{-1} with $a_{ii}(0)$ and $b_{ii}(0)$ different than zero. The controller that gives closed-loop transfer matrix $G_{CL}(z)$ is

$$G_C(z) = G(z)^{-1}\ G_{CL}(z) = (M(z)\ G(z))^{-1}$$

Since this must be proper and since $(\xi_G^{\ell}(z)G(z))$ is so, so must be their product $\xi_G^{\ell}(z)M(z)^{-1}$. Then from (3-15) follows that $\xi_G^{\ell}(z)M(z)^{-1}$ is lower triangular with diagonal elements

$$(G_{CL}(z))_{ii} = z^{-m_{ii}} \frac{b_{ii}(z^{-1})}{a_{ii}(z^{-1})}\ ; \qquad m_{ii} \geqslant k_i$$

which implies that a step change in y_{si} has minimum delay k_i and therefore

$$t_i \geqslant t_* + k_i\ ; \qquad i = 1,2,\ldots,\ell \qquad (3\text{-}16)$$

The IM controller (3-13) gives closed loop transfer matrix

$$G_{CL}(z) = \xi_G^{\ell}(z)^{-1}\ \xi_G^{\ell}(1) = \begin{bmatrix} z^{-k_1} & 0 & \ldots & 0 \\ a'_{21}(z)z^{-k_2} & z^{-k_2} & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ a'_{\ell 1}(z)z^{-k_\ell} & a'_{\ell 2}(z)z^{-k_\ell} & \ldots & z^{-k_\ell} \end{bmatrix}$$

where $a'_{ij}(z)$ are polynomials in z with $a'_{ij}(1) = 0$. From the above equation it follows that for every possible set-point change at t_*

$$y_i(t_*+k_i) = y_{si} \;; \qquad i = 1,2,\ldots,\ell$$

and thus (3-13) is the optimal controller.

(ii) From the argument above it follows immediately that the minimal times t_i are equal to t_*+k_i.

(iii) Let the ij element of the plant transfer matrix be

$$(G(z))_{ij} = z^{-d_{ij}} \frac{\sum_{k=0}^{b} b_{ijk} z^{-k}}{\sum_{k=0}^{a} a_{ijk} z^{-k}}$$

and let $d_i = \min_j (d_{ij})$. It is clearly impossible to influence output y_i before time t_*+d_i. But if the interactor is diagonal, $k_i = d_i$ and from part (ii) of the theorem, it follows that

$$y_i(t) = y_{is} \text{ (or } e_i(t) = \text{) for } t \geqslant t_*+k_i$$

The decoupling property follows immediately from the fact that

$$G_{CL}(z) = \xi_G^{\ell}(z)^{-1} \xi_G^{\ell}(1)$$

(iv) The criterion requires that $J_1 = \sum_{t=t_*}^{\infty} e_1^2(t)$ must be minimized; if this minimum is zero (in which case $y_{1s} = 0$, i.e., no set-point change in y_1 took place) $J_2 = \sum_{t=t_*}^{\infty} e_2^2(t)$

must be minimized (while maintaining $e_1(t) = 0$); if both minima are zero, in which case $y_{1s} = y_{2s} = 0$, $J_3 = \sum_{t=t_*}^{\infty} e_3^2(t)$ must be minimized (while maintaining $e_1(t) = e_2(t) = 0$), and so on.

Clearly, a set-point change in y_i should not affect an output with a lower index and thus the optimal closed loop transfer matrix is lower triangular. From (i) we have that for a lower triangular G_{CL} the minimum delay at each diagonal element is k_i. Controller (3-13) sets $y_i(t) = y_{is}$ for $t \geqslant t_*+k_i$. Clearly, this minimizes J_1. If the resulting $e_1(t) = 0$ for all t_1, in which case $y_{1s} = 0$, $e_2(t)$ depends only on the second diagonal element of G_{CL} and, since (3-13) gives $e_2 = 0$ at the first time it can be influenced, we conclude that (3-13) minimizes J_2. Similarly, one proves that if $\min_{i<j} J_i = 0$, controller (3-13) minimizes J_j. This completes the proof.

Theorem 3.8 (ii) provides a useful and convenient way to determine whether the optimal closed loop transfer matrix can be diagonal. In particular, the necessary and sufficient condition is $\det[(\operatorname{diag} z^{d_i})G(z)] \neq 0$ for $z = \infty$ where $d_i = \min_j d_{ij}$.

Theorem 3.9. (Characterization of all closed loop transfer matrices.) Suppose that the closed loop transfer matrix is desired to be of the form $G_d(z)$. Then there exists a stable internal model controller which will achieve this if and only if $\xi_G^{\ell} \, \xi_{G_d}^{\ell^{-1}}$ is proper.

Proof: The proof follows directly from Theorem 3.5.

An example is given next. Let

$$G(z) = \frac{1}{1-0.5\ z^{-1}} \begin{bmatrix} 2z^{-3} & z^{-5} \\ z^{-2} & z^{-3} \end{bmatrix}$$

We have $d_1 = 3$, $d_2 = 2$ and

$$\det \begin{bmatrix} z^3 & 0 \\ 0 & z^2 \end{bmatrix} G(z) \to 0 \text{ as } z\to\infty .$$

Therefore, any decoupled closed loop response is not optimal. By the constructive procedure in Appendix B, we find that

$$\xi_G^{\ell}(z) = \begin{bmatrix} z^3 & 0 \\ -\frac{1}{2}z^4 & z^3 \end{bmatrix}$$

Indeed, $\lim_{z\to\infty} \xi_G^{\ell}(z)G(z) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, which is nonsingular. Then the internal model controller given by Theorem 3.8 as

$$G_C(z) = \begin{bmatrix} \frac{1}{2} - \frac{1}{4}z^{-1} - \frac{1}{4}z^{-3} & \frac{z^{-3}}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

will give the closed loop transfer function

$$G_{CL}(z) = \begin{bmatrix} z^{-3} & 0 \\ \frac{1}{2}(z^{-2}-z^{-3}) & z^{-3} \end{bmatrix}$$

which is optimal.

Assume now that a decoupled closed loop response is desired, i.e., $G_d(z) = \text{diag}[z^{-p_1}\ z^{-p_2}]$. Then the best possible proper internal model controller, i.e., the one leading to the smallest possible p_1, p_2, can be determined by Theorem 3.9. For the previous example,

$$\xi_G^\ell \, \xi_{G_d}^{\ell-1} = \begin{bmatrix} z^{3-p_1} & 0 \\ -\frac{1}{2} z^{4-p_1} & z^{3-p_2} \end{bmatrix}$$

must be proper, i.e., $p_1 \geqslant 4$ and $p_2 \geqslant 3$. Then the optimal G_C corresponds to $p_1 = 4$, $p_2 = 3$ and is given by

$$G_C = G^{-1} G_d = \frac{1}{2} \begin{bmatrix} z^{-1} & -z^{-2} \\ -1 & 2 \end{bmatrix}$$

It is clear that this can be generalized to

Theorem 3.10. If a diagonal closed loop transfer matrix is desired, i.e., $G_s = \text{diag}\ (z^{-p_i})$, then the smallest possible p_i, $i = 1,2,\ldots,\ell$ are given by $p_i = \partial C_i(\xi_G^\ell)$ = ith column degree of ξ_G^ℓ. The column degrees of the left interactor will be the smallest possible deadtimes for a diagonal G_d. Any G_d with a smaller p_i will require an improper G_C.

Disturbance Estimation via the Right Interactor

Consider again the configuration of Figure (3-2). Then in a similar way to Theorem 3.8 we have

Theorem 3.11. The best possible E is given by

$$E(z) = \xi_G^r(1) \; \xi_G^r(z)^{-1} \; G(z)^{-1}$$

where ξ_G^r is the right interactor and G has no finite unstable zeros.

Theorem 3.11 states that only the part ξ_G^{r-1} d of the disturbance d is available for further use, e.g., for incorporation in the control law for disturbance rejection. All admissible estimators are given by

$$T\,\xi_G^{r-1}\,G^{-1}$$

where T is stable and proper.

Adaptive Controller Design via the Right Interactor

Lemma 3.12. For any strictly proper transfer function $G(z)$, there exist left coprime polynomial matrices $A'(z)$, $B'(z)$ such that

(i) $G(z)\,\xi_G^r(z) = A'(z)^{-1}B'(z)$

where $A'(z)$ is row reduced (22).

(ii) If a_i denotes the ith row degree of $A'(z)$ and b_i denotes the ith row degree of $B'(z)$, then $a_i = b_i$ and $B'(z)$ is row reduced.

Proof:

(i) See (22), (23).

(ii) Let A'_o, B'_o denote the matrices with elements at each row equal to the coefficients of the terms z^{a_i}, z^{b_i} in $A'(z)$, $B'(z)$, respectively. We then have

$$\det(G(z)\,\xi_G^r(z)) = \frac{\det B'(z)}{\det A'(z)} = \frac{\det B'_o z^b + \text{l.o.t.}}{\det A'_o z^a + \text{l.o.t.}}$$

where $a = \sum_{i=1}^{\ell} a_i$, $b = \sum_{i=1}^{\ell} b_i$, and l.o.t. stands for lower order terms. Now since $G(z)\,\xi_G^r(z)$ is proper and $A'(z)$ is row reduced from (22) $a_i \geq b_i$. Then $\lim_{z\to\infty} \det(G(z)\,\xi_G^r(z)) = \det \lim_{z\to\infty}(G(z)\,\xi_G^r(z))$ is nonsingular if and only if $a = b$ (i.e., $a_i = b_i$, $i = 1,2,\ldots,\ell$) and $\det B'_o \neq 0$. Q.E.D.

Theorem 3.13. Let $G(z)$ be strictly proper and d = maximum degree in any element of $\xi_G^r(z)$ and m = minimum degree in any element of $\xi_G^r(z)$. Also let $v(z) = z^d \, \xi_G^r(z)^{-1} u(z)$. Then

(i) If $v(t)$ is determined at t, $u(t)$ can be determined from $v(t)$ after an initialization time of $d-m$ sampling intervals.

(ii) There exist polynomial matrices $A(q^{-1})$, $B(q^{-1})$ in the backward shift operator q^{-1} such that

$$A(q^{-1})y(t+d) = B(q^{-1})v(t) \tag{3-17}$$

with $A(0) = I$ and $B(0)$ nonsingular.

Proof:

(i) We have that

$$u(t) = q^{-d} \, \xi_G^r(q)v(t)$$

and thus $u(t)$ contains components of $v(t), v(t-1), \ldots, v(t+m-d)$. The latter are all available after $d-m$ initial steps. Finally, the fact that $q^{-d} \, \xi_G^r(q)$ is causal allows for determination of $u(t)$ at time t.

(ii) Let

$$T(z) = \mathrm{diag}(z^{a_i})$$

Then (3-17) follows with

$$A(q^{-1}) = (T(q)A_o')^{-1} \, A'(q)$$

$$B(q^{-1}) = (T(q)A_o')^{-1} \, B'(q)$$

Now that $B(0)$ is nonsingular, an adaptive controller can be designed as in section (3.2). Model (3-17) is written in the predictor form

$$y(t+d) = C(q^{-1})y(t)+D(q^{-1})v(t)$$

The control law based on the minimization of the performance index

$$J = \| P(q^{-1})e(t+d) \|^2 + \| Q'(q^{-1})\bar{v}(t) \|^2$$

where $e(t) = y(t)-y_s$, y_s is the set point, $\bar{v}(t) = v(t)-u_s$, and u_s is the steady state control corresponding to y_s, will give the control policy

$$P(q^{-1})(\hat{C}(q^{-1})y(t)+\hat{D}(q^{-1})v(t))+Q(q^{-1})\bar{v}(t) = P(1)y_s \tag{3-18}$$

where ^ denotes estimate at time t. Assuming that $\xi_G^r(z)$ is known, the v(t) given by (3-18) will give u(t) by Theorem 3.13 (i). It is noted that a similar theorem was given in Goodwin and Long (25) via the $\xi_G^\ell(z)$. The following points out the usefulness of Theorem 3.13 and the above suggested algorithm. Knowledge of a non-diagonal left interactor matrix requires knowledge of some of the systems parameters. For a large class of systems with a non-diagonal left interactor the right interactor is either diagonal or requires knowledge of fewer system parameters. It is therefore more suitable for adaptive controller design. For instance, consider the strictly proper transfer function

$$G(z) = \begin{bmatrix} b_1 z^{-1} & b_2 z^{-5} \\ b_3 z^{-2} & b_4 z^{-4} \end{bmatrix} \qquad b_1 b_2 b_3 b_4 \neq 0$$

for which it is found readily that

$$\xi_G^{\ell}(z) = \begin{bmatrix} z & 0 \\ -\frac{b_3}{b_1} z^3 & z^4 \end{bmatrix}$$

and

$$\xi_G^{r}(z) = \begin{bmatrix} z & 0 \\ 0 & z^4 \end{bmatrix}$$

Design of an adaptive controller as in Goodwin and Long (25) requires knowledge of b_3/b_1. This is not required for controller design via the $\xi_G^r(z)$. It is finally noted that no diagonal form of $A(q^{-1})$ is required here as in Goodwin et al. (15) or as in the adaptive controller suggested in section 3.2 and, therefore, redundant parametrizations are avoided.

Generalized Interactors

The following theory is a generalization of the results in the previous sections and allows for determination of the invariant process characteristics which are associated with the finite-unstable-zeros structure of the system.

Lemma 3.14. Let $s, |s| \neq 1$ be an unstable zero of the transfer function $G(z)$ and assume that $G(z)$ is full rank and proper at s, i.e., $\lim_{z \to s} G(z) < \infty$. Then there exists a unique lower triangular rational polynomial matrix $\xi_{G,s}^{\ell}(z)$ and a unique upper triangular rational polynomial matrix $\xi_{G,s}^{r}(z)$ of the form

$$\xi^{\ell}_{G,s}(z) = H^{\ell}_{G,s}\left(\frac{1-sz}{z-s}\right)\mathrm{diag}\left[\left(\frac{1-sz}{z-s}\right)^{-k_i}\right]$$

$$\xi^{r}_{G,s}(z) = \mathrm{diag}\left[\left(\frac{1-sz}{z-s}\right)^{-r_i}\right]H^{r}_{G,s}(z)$$

$$(H^{\ell}_{G,s}(z))_{ii} = (H^{r}_{G,s}(z))_{ii} = 1$$

with $H^{\ell}_{G,s}(z)$ lower triangular, $H^{r}_{G,s}(z)$ upper triangular, and $h^{\ell}_{ij}(z)$, $h^{r}_{ij}(z)$ both multiples of $\frac{1-sz}{z-s}$ or zero, which are called left and right s-interactors of G(z) and which have the following properties

(i) $\lim_{z\to s} \xi^{\ell}_{G,s}(z)\ G(z) = K^{\ell}_{G}$ is nonsingular

$\lim_{z\to s} G(z)\ \xi^{r}_{G,s}(z) = K^{r}_{G}$ is nonsingular

(ii) $\xi^{\ell}_{G,s}(z)$ and $\xi^{r}_{G,s}(z)$ are both stably invertible.

Proof: Let $z = \frac{1+sw}{s+w}$ and apply Lemma 3.3 for $G'(w)\ (= G(\frac{1+sw}{w+s}))$.

Note also that the s-interactors can be constructed as in Appendix B in the w domain and then transformed to the z-domain by $w = \frac{1-sz}{z-s}$.

Remark: If $|s| = 1$, then instead of $\frac{1-sz}{z-s}$ in the above Lemma, we will have $\frac{1-(s+\varepsilon)z}{z-s}\ \frac{1-s}{1-(s+\varepsilon)}$ where $|\varepsilon|$ is small. This modification is necessary in order for the s interactor to be stably invertible without pole-zero cancellations on the unit circle.

Similarly, one can show

Lemma 3.15. If G(z) is s-proper, then $G(z)^{-1}\ \xi^{\ell}_{G,s}(z)^{-1}$ and $\xi^{r}_{G,s}(z)^{-1}\ G(z)^{-1}$ are s-proper.

Theorem 3.16. Let $G_1(z)$, $G_2(z)$ be s-proper. Then there exists an s-proper $G(z)$ such that

(i) $G_1(z)\ G(z) = G_2(z)$ if and only if $\xi^{\ell}_{G_1,s}(z)\ \xi^{\ell}_{G_2,s}(z)^{-1}$ is s-proper and

(ii) $G(z)\ G_1(z) = G_2(z)$ if and only if $\xi^{r}_{G_2,s}(z)^{-1}\ \xi^{r}_{G_1,s}(z)$ is s-proper.

Corollary 3.17. If $G_1(z)$ is s-proper, then

(i) $G_1(z)$ has an s-proper right inverse if and only if $\xi^{\ell}_{G_1,s}(z) = I$ and

(ii) $G_1(z)$ has an s-proper left inverse if and only if $\xi^{r}_{G_1,s}(z) = I$.

Next, define S to be the set of all unstable zeros of $G(z)$, both finite and at infinity, (time delays), i.e., $S = \{s_i/i = 1,2,\ldots,N\}$, $s_N = \infty$. Also define the generalized left interactor to be

$$\bar{\xi}^{\ell}_G(z) = \prod_{i=1}^{N} \xi_i(z)$$

where $K_0(z) = G(z)$, $K_i(z) = \xi^{\ell}_i(z)K_{i-1}(z)$, and $\xi_i(z)$ is the left s_i-interactor of $K_{i-1}(z)$.

Theorem 3.18. The rational matrix $\bar{\xi}_G(z)$ is an S-interactor of $G(z)$.

Proof: Let s_i belong to S. Then

$$\lim_{z\to s_i} \bar{\xi}_G(z)\ G(z) = \lim_{z\to s_i} (\xi_N(z)\ldots\xi_{i+1}(z))(\xi_i(z)\ K_{i-1}(z))$$

which is nonsingular since both

$$\lim_{z\to s_i} (\xi_N(z)\ldots\xi_{i+1}(z)) \quad \text{and} \quad \lim_{z\to s_i} (\xi_i(z)\, K_{i-1}(z)) \quad \text{are so.}$$

Similarly, one can show the corresponding theorem to Theorem 3.18 for the right generalized interactor and then Lemma 3.4, Theorem 3.5, and Corollary 3.6 are generalized to

Lemma 3.19. If $G(z)$ is S-proper, then $G(z)^{-1}\ \bar{\xi}_G^{\ell}(z)^{-1}$ and $\bar{\xi}_G^{r}(z)^{-1}\ G(z)^{-1}$ are S-proper.

Theorem 3.20. Let $G_1(z)$, $G_2(z)$ be S-proper. Then there exists an S-proper $G(z)$ such that

(i) $G_1(z)\ G(z) = G_2(z)$ if and only if $\bar{\xi}_{G_1}^{\ell}(z)\ \bar{\xi}_{G_2}^{\ell}(z)^{-1}$ is S-proper.

(ii) $G(z)\ G_1(z) = G_2(z)$ if and only if $\bar{\xi}_{G_2}^{r}(z)^{-1}\ \bar{\xi}_{G_1}^{r}(z)$ is S-proper.

Corollary 3.21. If $G_1(z)$ is S-proper, then

(i) $G_1(z)$ has an S-proper right inverse if and only if $\bar{\xi}_G^{\ell}(z) = I$ and

(ii) $G_1(z)$ has an S-proper left inverse if and only if $\bar{\xi}_G^{r}(z) = I$.

The above results can be used in the design of internal model controllers for open loop stable systems with multiple delays and unstable zeros. In particular, Theorems 3.7, 3.8, 3.9, and 3.10 are generalized as follows.

Theorem 3.22. (Generalization of Theorem 3.7). Let $G(z)$ be a stable transfer matrix. Then for any proper and stable transfer matrix T with $T(1) = I$ the IM controller

$$G_C(z) = G(z)^{-1}\ \bar{\xi}_G^{\ell}(z)^{-1}\ \bar{\xi}_G^{\ell}(1)\ T(z)$$

is both proper and stable and eliminates steady state offset.

Theorem 3.23 (Generalization of Theorem 3.8). If $G(z)$ is stable, then

(i) the optimal IM controller with respect to the SSE of Theorem 3.8 (iv) is given by

$$G_C(z) = G(z)^{-1} \, \bar{\xi}_G^{\ell}(z)^{-1} \, \bar{\xi}_G^{\ell}(1) \tag{3-19}$$

and results in lower triangular closed loop transfer matrix. This is proved using the methodology of Theorem 3.8 and the fact that J_1 is minimized if the closed loop transfer matrix has a pole at $\frac{1}{s_i}$ where s_i is a finite unstable zero (6).

(ii) The optimal IM controller (3-19) provides complete decoupling if and only if $\bar{\xi}_G^{\ell}(z)$ is diagonal.

Theorem 3.24. (Generalization of Theorem 3.9). If G_d is a desired closed loop transfer matrix, then it is achievable by the IM controller $G_C = G^{-1} G_d$ if and only if $\bar{\xi}_G^{\ell} \, \bar{\xi}_G^{\ell -1}$ is stable and proper.

Theorem 3.25. (Generalization of Theorem 3.10). If a diagonal closed loop transfer matrix is desired, $(G_s)_{jj} = \left(\prod_{i=1}^{n_j} \left(\frac{z-s_i}{1-s_i z}\right)^{p_i} \right)$ then the smallest possible p_i, $i = k, 2, \ldots \ell$ is given by the highest power of $\frac{z-s_i}{1-s_i z}$ in the jth column of $\bar{\xi}_G^{\ell}$.

An Optimal Feedforward Disturbance Estimator

The design of a disturbance estimator via the right generalized interactor is based on

Theorem 3.26. The estimator E in Figure 3-2 given by

$$E(z) = T(z) \, \bar{\xi}_G^{r}(1) \, \bar{\xi}_G^{r}(z)^{-1} \, G(z)^{-1}$$

where T is any stable and proper transfer function with

$T(1) = 1$ and where $\bar{\xi}_G^r$ is the right generalized interactor of G, is both proper and stable.

Proof: This theorem is dual to Theorem 3.22.

An example is given next. Let

$$G(z) = \begin{bmatrix} \frac{.6}{z-.4} & \frac{.5}{z-.5} \\ \frac{.6}{z-.5} & \frac{.6}{z-.4} \end{bmatrix}$$

This transfer matrix has unstable zeros at $1+(.3)^{1/2}$ $(= s_1)$ and at $\infty (= s_2)$. We find

$$\xi_{G,s_1}^{\ell}(z) = \begin{bmatrix} 1 & 0 \\ -\alpha \frac{1-s_1 z}{z-s_1} & \frac{1-s_1 z}{z-s_1} \end{bmatrix}$$

where $\alpha = (.6+(.3)^{1/2})/(.5+(.3)^{1/2})$. Next, letting

$$\xi_{G,s_1}^{\ell}(z)\ G(z) = K_1(z)$$

we find

$$\xi_{K_1,s_2}^{\ell}(z) = \begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix}$$

Then

$$\bar{\xi}_G^{\ell}(z) = \xi_{K_1,s_2}^{\ell}(z)\ \xi_{G,s_1}^{\ell}(z) = \begin{bmatrix} z & 0 \\ -\alpha z\ \frac{1-sz}{z-s} & z\ \frac{1-sz}{z-s} \end{bmatrix}$$

and the closed loop transfer matrix is given by

$$G_{CL}(z) = z^{-1} \begin{bmatrix} 1 & 0 \\ \alpha \dfrac{(s_1+1)(1-z)}{1-s_1 z} & \dfrac{z-s_1}{1-s_1 z} \end{bmatrix}$$

The IM controller given by (3-19) is strictly proper and has poles at $1/s_1$ and .4, i.e., it is both implementable and stable.

CHAPTER IV
ASYMPTOTIC STABILITY OF SELF-TUNING CONTROLLERS

4.1 Introduction

The global asymptotic stability of adaptive controllers applied to linear systems has remained a long-standing open question during the last decade due to the implicit time varying nature of the closed loop. The problem was first addressed by Parks (32), where the design was based on Lyapunov's second method. Feuer and Morse (33) suggested a stable algorithm which was, however, technically involved. Landau (34) used Popov's hyperstability theory to prove the stability of some continuous and discrete-time model reference adaptive schemes. Also Narendra and Lin (35) established the stability of the discrete version of the algorithm of Narendra and Valavani (36). Goodwin, Ramadge, and Caines (15) proved the stability of minimum variance adaptive algorithms applied to minimum phase discrete-time linear systems. Cordero and Mayne (37) and Ydstie and Sargent (38) extended these results to single-input single-output ST regulators employing a variable forgetting factor.

In the stochastic case, Ljung (39, 40) presented a technique for analyzing the convergence of stochastic self-tuning (ST) regulators. Conditioned stability proofs for stochastic algorithms were also given by Goodwin, Ramadge, and Caines

(41) and Sin and Goodwin (42). Landau (43) used a martingale type approach to establish boundedness of the variables as well as asymptotic properties. Asymptotic convergence was also established for an adaptive pole placement algorithm in Goodwin and Sin (44). More recently, stability issues of a multivariable adaptive pole placement algorithm were discussed in Elliot et al. (45).

In 1975, Clarke and Gawthrop (21, 46) developed the self-tuning controller. This algorithm not only does penalize output error as does the classical ST regulator (mimimun variance), but can also penalize excessive control fluctuation and it follows set-point changes. The controller is based on a cost function with á priori chosen constant weights. If this cost function is dictated by economic considerations, controller design is easier than in the case of adaptive pole placement controllers, which can be shown to involve time-varying weights (Goodwin and Sin (44) and Kumar and Moore (47)). Another advantage of a constant weights self-tuning controller is its simplicity. The control law is explicit and does not require the solution of Diophantine equations on line, thus avoiding the associated ill-conditioning problems which might arise in adaptive pole placement algorithms (Kumar and Moore (47)).

A first stability analysis for self-tuning controllers of the Clarke-Gawthrop type was given in Gawthrop (48), with the conclusion that the algorithm needed to be modified in order to guarantee convergence. Gawthrop and Lim (2) pointed out the enhanced robustness characteristics of the self-tuning controller in the presence of plant nonlinearities, unmodelled disturbances, and plant-model order mismatch. An overview of the recent results was given in Gawthrop (49). However, the asymptotic convergence of the self-tuning controller has not been established yet (Kumar and Moore, (47), Ydstie and Liu (50)), albeit the fact that it constitutes one of the few viable adaptive controllers for nonminimum phase systems (Clarke, (3)).

This section establishes the asymptotic stability of single-input single-output (SISO) and multiple-input multiple-output (MIMO) STCs of the above type, i.e., based on the minimization of a quadratic performance index which includes penalization of both output and input deviation as well as velocity. First, some properties of the most widely used parameter estimators are stated. Then a stability proof is given for a modified version of the MIMO STC of equation (2-21) applied to minimum phase systems.

4.2 Properties of the Parameter Estimators

1. $\lim_{t\to\infty} \nu_j(t) = 0$ where $\nu_j(t) = \dfrac{\varepsilon_j(t)}{(s+g(t+1))^{1/2}}$,

   $g(t-1) = x'(t-1)F(t-1)x(t-1)$, F is the covariance matrix (see Appendix A) and s = 1 or s = $\lambda(t)$ (the forgetting factor) depending on the estimator.

2. $\lim_{t\to\infty} \tilde{\theta}(t) = \tilde{\theta}_\infty$ and $\|\tilde{\theta}(t)\| \leqslant \|\tilde{\theta}(t-1)\| \leqslant \|\tilde{\theta}(0)\|$

where $\tilde{\theta}(t) = \theta-\hat{\theta}(t)$ and $\theta = \begin{bmatrix} \theta_1' \\ \theta_2' \\ \cdot \\ \cdot \\ \cdot \\ \theta_\ell' \end{bmatrix}$

3. $\| F(t) \| < C < \infty$ and $\lim_{t\to\infty} F(t) = F_\infty$; C, F_∞ are constants.

4. $|g(t-1)| < C\| x(t-1) \|^2$

5. $\lim_{t\to\infty} H(t) = 0$ where $H(t) = \tilde{\theta}(t-d)-\tilde{\theta}(t)$

The above properties were established in (15), (31), (38), (41), (51), and (52).

4.3 Some Technical Lemmas

The following lemmas will be needed further:

Lemma 4.1. Let $p(t)$ and $q(t)$ be real vector sequences related by

$$G_1(q^{-1})p(t) = G_2(q^{-1})q(t)$$

where $G_1(q^{-1})$ and $G_2(q^{-1})$ are matrix polynomials in the backward shift operator q^{-1}, and $G_1(q^{-1})$ is stably invertible, that is, $\det(G_1(z^{-1})) \neq 0$ for $|z| \geqslant 1$. Then

(i) there exist nonnegative constants k_1 and k_2 such that

$$\| p(t) \| \leqslant k_1+k_2 \max_{0\leqslant r\leqslant t} \| q(r) \| \qquad (4\text{-}1)$$

(ii) if $\lim_{t\to\infty} q(t) = 0$, then $\lim_{t\to\infty} p(t) = 0$ $\qquad$ (4-2)

Proof: The results are standard if the system is viewed as a forced linear system with input $q(t)$ and output $p(t)$.

Lemma 4.2. Let $p(t)$, $q(t)$ be sequences in R^{ℓ}, R^{m}, respectively and let $G(t)$ be an $\ell \times m$ real matrix with $\lim_{t\to\infty} G(t) = 0$. If there exist nonnegative constants k_1 and k_2 such that

$$\| q(t) \| \leqslant k_1 + k_2 \max_{0\leqslant\tau\leqslant t} \| p(\tau) + G(r) q(\tau) \| \tag{4-3}$$

then there exist nonnegative constants k_3, k_4 such that

$$\| q(t) \| \leqslant k_3 + k_4 \max_{0\leqslant\tau\leqslant t} \| p(\tau) \|$$

Proof: Inequality (4-3) implies that for large enough t

$$\begin{aligned} \max_{0\leqslant\tau\leqslant t} \| q(\tau) \| \leqslant k_1 + k_2 \max_{0\leqslant\tau\leqslant t} \| p(\tau) \| &+ \\ k_2 \max_{0\leqslant\tau\leqslant T} [\| G(\tau) \| \, \| q(\tau) \|] &+ \\ k_2 \max_{T\leqslant\tau\leqslant t} [\| G(\tau) \| \, \| q(\tau) \|] & \end{aligned} \tag{4-4}$$

where $T<\infty$ is fixed and such that $\| G(t) \| \leqslant 1/2k_2$ for $t>T$. Such a time T exists since $\lim_{t\to\infty} G(t) = 0$. From inequality (4-4) it follows that

$$\begin{aligned} \max_{T\leqslant\tau\leqslant t} \| q(\tau) \| \leqslant k_1 + k_2 \max_{0\leqslant\tau\leqslant T} [\| G(\tau) \| \, \| q(\tau) \|] &+ \\ k_2 \max_{0\leqslant\tau\leqslant t} \| p(\tau) \| + 1/2 \max_{T\leqslant\tau\leqslant t} \| q(\tau) \| & \end{aligned} \tag{4-5}$$

or

$$1/2 \max_{T\leqslant\tau\leqslant t} \| q(\tau) \| \leqslant M+k_2 \max_{0\leqslant\tau\leqslant t} \| p(\tau) \| \tag{4-6}$$

where

$$M = k_1+k_2 \max_{0\leqslant\tau\leqslant T} [\| G(\tau) \| \; \| q(\tau) \|] < \infty \tag{4-7}$$

From the above inequality, one obtains the result with $k_3 = 2M+N$ and $k_4 = 2k_2$, where $N = \max\limits_{0\leqslant\tau\leqslant T} \| q(\tau) \| < \infty$.

Lemma 4.3. If r(t), s(t) are real scalar sequences and p(t), q(t) are sequences in R^ℓ satisfying

$$\lim_{t\to\infty} \frac{\| p(t) \|^2}{r(t)+s(t) \| q(t) \|^2} = 0 \tag{4-8}$$

then subject to

(i) $0 < r(t) \leqslant k_1,\ 0 \leqslant s(t) \leqslant k_2,\ k_1 < \infty,\ k_2 < \infty$ (4-9)

(ii) $\| q(t) \| \leqslant c_1+c_2 \max\limits_{0\leqslant r\leqslant t} \| p(r) \|,\ 0 < c_1 < \infty,\ 0 < c_2 < \infty$ (4-10)

it follows that $\lim\limits_{t\to\infty} p(t) = 0$.

Proof: See Goodwin et al. (15).

4.4 Asymptotic Stability of Self-Tuning Controllers for Linear Minimum-Phase Systems with Multiple Delays

It is established here that a class of quadratic STCs achieves asymptotically the control objective for any initial conditions of a linear plant and/or the estimator, while all process variables remain bounded.

Consider the system (2-16) with ℓ=m and the performance measure (2-20). The optimal controller with respect to (2-21) is given by

$$D^{(0)\prime}Q_1\ e(t+d+1)+Q_2\bar{v}(t)+Q_3(\bar{v}(t)-\bar{v}(t-1)) = 0 \qquad (4\text{-}11)$$

or, assuming that $D^{(0)\prime}Q_1$ is invertible,

$$\bar{e}(t+d+1)+Q(q^{-1})\bar{v}(t) = 0$$

where

$$Q(q^{-1}) = Q_4+Q_5(1-q^{-1});\ Q_4 = (D^{(0)\prime}Q_1)^{-1}Q_2,$$

$$Q_5 = (D^{(0)\prime}Q_1)^{-1}Q_3$$

One can now choose Q_4, Q_5 if an á priori estimate of $D^{(0)}$ is available. Guidelines for obtaining a $D^{(0)}$ estimate are given in Peltomaa and Koivo (53).

The above modification makes the control law linear with respect to the estimated parameters. Since $e(t+d+1)$ is unknown at time t, it is substituted by its estimate and the implementable control law is

$$\hat{e}(t+d+1)+Q(q^{-1})\bar{v}(t) = 0 \qquad (4\text{-}12)$$

where

$$\hat{e}(t+d+1) = \hat{\theta}(t)x(t+d)-y_s$$

Suppose now that the direct estimation scheme is used to estimate the parameters of (4.12).

Lemma 4.4. The following are true

(i) $\varepsilon(t) = \tilde{\theta}(t-1)x(t-1)$

where

$$\varepsilon'(t) = [\varepsilon_1(t), \varepsilon_2(t), \ldots, \varepsilon_\ell(t)]$$

$$\varepsilon_j(t) = y_j(t+k_j-k) - \hat{\theta}_j(t-1)x(t-1)$$

(ii) $e(t) = \tilde{\theta}(t-d-1)x(t-1) - Q(q^{-1})\bar{v}(t-d-1)$

(iii) $e(t) = \varepsilon(t) + H(t-1)x(t-1) - Q(q^{-1})\bar{v}(t-d-1)$ (4-13)

where

$$H(t-1) = \tilde{\theta}(t-d-1) - \tilde{\theta}(t-1)$$

Proof:

(i) From (2-22) and the definition of $\varepsilon_j(t)$, (i) follows readily.

(ii) From the control law (4-12), $\bar{v}(t-d-1)$ is such that

$$\hat{\theta}(t-d-1)x(t-1) + Q(q^{-1})\bar{v}(t-d-1) = y_s$$

From the definition of e(t), it also follows that

$$e(t) = \theta x(t-1) - y_s$$

Combining the above two equations, one obtains the result.

(iii) Multiplication of H(t-1) on the right by x(t-1) and use of (i) and (ii) yield the result

Lemma 4.5. The tracking error dynamics are described by

$$G_1(q^{-1})e(t) = G_2(q^{-1})[\varepsilon(t) + H(t-1)x(t-1)] \quad (4\text{-}14)$$

where $G_1(q^{-1})$, $G_2(q^{-1})$ are matrix polynomials in q^{-1} and $G_1(q^{-1})$ is stably invertible if and only if

$$G(q^{-1}) \triangleq B(q^{-1})+A(q^{-1})Q(q^{-1})$$

is stably invertible.

Proof. For $Q = 0$, (4-14) follows from (4-13) with $G_1(q^{-1}) = G_2(q^{-1}) = I$. Now let $Q \neq 0$. We are interested in the case where Q_4+Q_5 is nonsingular. Then from Gohberg et al. (54), there exist unique matrix polynomials $S(q^{-1})$ and T (= constant since degree of $Q(q^{-1})$ in q^{-1} is 1) such that

$$B(q^{-1}) = S(q^{-1})Q(q^{-1})+T \tag{4-15}$$

Specifically in this case,

$$T = \sum_{i=0}^{n} B^{(i)}((Q_4+Q_5)^{-1}Q_5)^{n-i}$$

which we assume to be nonsingular. Substituting now from (4-15) in the error form of model (2-16), namely

$$A(q^{-1})e(t) = B(q^{-1})\bar{v}(t-d-1)$$

and dropping the argument q^{-1} for convenience, one obtains after using (4-13)

$$Ae(t) = S[\epsilon(t)+H(t-1)x(t-1)-e(t)]+T\bar{v}(t-d-1) \tag{4-16}$$

Solving (4-16) for $\bar{v}(t-d-1)$ and substituting in (4-13) yields (4-14) with

$$G_2 = I+QT^{-1}S \tag{4-17}$$

$$G_1 = G_2+QT^{-1}A \tag{4-18}$$

It is seen that degree $(G_1(q^{-1})) = n+2$ and that degree $(G_2(q^{-1})) = n$. Finally, use of (4-15) in (4-18) results in

$$QT^{-1}G = G_1Q$$

It then follows that det $G_1 = c$ det G where $c = \det(T^{-1})$ is nonzero since T was assumed to be nonsingular. It follows that G_1 is stably invertible if and only if G is stably invertible.

The overall asymptotic stability of the closed loop is established next for minimum-phase systems. The proof given here is a generalization of the proof in Goodwin et al. (15), which concerned the case $Q_4 = Q_5 = 0$. The fact that the weights Q_4, Q_5 have to be chosen suitably is pointed out here.

Theorem 4.6. Under the assumption that det $B(z)+A(z)Q(z) \neq 0$, $|z| \leqslant 1$, the controller (4-12) when applied to a minimum-phase system (2-16) leads to

$$\lim_{t\to\infty} e(t) = 0$$

via a bounded sequence v(t).

The assumption deserves some comments. If the process is controlled by the optimal control law, then the closed loop system is described by

$$G_1(q^{-1})e(t) = 0$$

Hence, a necessary condition for stability is that det $G(z) \neq 0$, $|z| \leqslant 1$. It is also noted that for $Q_4 = Q_5 = 0$, the above assumption is vacuously true.

Proof: From (4-14), we have

$$G_1(q^{-1})e(t) = G_2(q^{-1})\varepsilon(t)+G_2(q^{-1})H(t-1)x(t-1)$$

Since the system is minimum phase, $B(q^{-1})$ is stably invertible and from Lemma 4.1 it follows that there exist nonnegative constants c_1', c_2' so that

$$\|\bar{v}(t-d-1)\| \leqslant c_1'+c_2' \max_{0\leqslant\tau\leqslant t} \|e(\tau)\|$$

from which one finds c_1, c_2 nonnegative such that

$$\|x(t-1)\| \leqslant c_1+c_2 \max_{0\leqslant\tau\leqslant t} \|e(\tau)\| \tag{4-19}$$

Then it also follows from properties 1 and 4 of the estimators that for some constant nonnegative c_3, c_4

$$\|\varepsilon(t)\| \leqslant (c_3+c_4 \max_{0\leqslant\tau\leqslant t} \|e(\tau)\|)\ \|\nu(t)\| \tag{4-20}$$

and (4-14) gives, in view of (4-19) and (4-20),

$$\|G_1(q^{-1})e(t)\| \leqslant \sum_{i=0}^{n} \|G_{2i}\|\ (c_3+c_4 \max_{0\leqslant\tau\leqslant t} \|e(\tau)\|)\,\|\nu(t-i)\| + \sum_{i=0}^{n} \|G_{2i}\|\ \|H(t-i-1)\|\ (c_1+c_2 \max_{0\leqslant\tau\leqslant t} \|e(\tau)\|) \tag{4-21}$$

Case I. $e(t)$ bounded, i.e., $\|e(t)\|$ M, M real nonnegative.

Then since $\lim_{t\to\infty} \nu(t) = \lim_{t\to\infty} H(t) = 0$, we get that $\lim_{t\to\infty} \|G_1(q^{-1})e(t)\| = 0$, which in view of Lemma 4.1 (ii) yields $\lim_{t\to\infty} e(t) = 0$.

Case II. e(t) unbounded.

Let $G_1(q^{-1})e(t) = p(t)$. Then from Lemma 4.1, p(t) is also unbounded, i.e., there exists subsequence $\{t_k\}$ along which

$$\|p(t_k)\| = \max_{0\leqslant\tau\leqslant t_k} \|p(\tau)\| \quad \text{and} \quad \lim_{k\to\infty} \|p(t_k)\| = \infty \tag{4-22}$$

Also, since G_1 is assumed to be stably invertible by Lemma 4.1, there exist nonnegative constants k_1, k_2 such that

$$\max_{0\leqslant\tau\leqslant t} \|e(\tau)\| \leqslant k_1+k_2 \max_{0\leqslant\tau\leqslant t} \|p(\tau)\|$$

Then (4-21) becomes

$$\begin{aligned}\|p(\tau)\| \leqslant &\{C_3+C_4\{k_1+k_2 \max_{0\leqslant\tau\leqslant t} \|p(\tau)\|\}\}b(t) + \\ &\{C_1+C_2\{k_1+k_2 \max_{0\leqslant\tau\leqslant t} \|p(\tau)\|\}\}c(t)\end{aligned} \tag{4-23}$$

where

$$b(t) = \sum_{i=0}^{n} \|G_{2i}\| \, \|\nu(t-i)\|$$

$$c(t) = \sum_{i=0}^{n} \|G_{2i}\| \, \|H(t-i-1)\|$$

and

$$\lim_{t\to\infty} b(t) = \lim_{t\to\infty} c(t) = 0 \tag{4-24}$$

Considering now (4-23) along $\{t_k\}$ and for large enough t_k we have

$$\begin{aligned}0 \leqslant \|p(t_k)\| \{1-C_4k_2b(t_k)-C_2k_2c(t_k)\} \leqslant (C_3+C_4k_1)b(t_k) + \\ (C_1+C_2k_1)c(t_k)\end{aligned}$$

and then (4-24) forces

$$\lim_{k\to\infty} \| p(t_k) \| \{1-C_4k_2b(t_k)-C_2k_2c(t_k)\} = 0$$

However, by directly calculating this limit using (4-22), one finds it equal to $\infty \cdot 1 = \infty$, a contradiction. Hence, Case II is impossible and the result follows from Case I. Consequently, (4-19) implied the boundedness of $v(t)$.

4.5 Asymptotic Convergence of Self-Tuning Controllers

Asymptotic stability is established for linear, minimum, and nonminimum phase systems controlled by STCs. The latter are based on generalized cost function criteria in which the weights are polynomial matrices in q^{-1}. The systems can have an arbitrary input-output delay structure.

Self-Tuning Controllers Based on Measurement-Control Delay Factorization

The system is assumed to be described by (3-4) and to accept the factorization which transforms it to the form (3-6). The control law is given by (3-12).

For estimating the controller parameters, model (3-6) is rewritten as

$$y_j(t) = x_j'(t-1)\theta_j$$

where

$$x_j'(t-1) = [1\ y_j(t-d-1)\ldots y_j(t-d-x-1)\ v'(t-r-k_j-1)\ldots$$
$$v'(t-r-d-k_j-1)]$$

$$\theta_j' = [s_j^*\ C_{jj}^{(0)}\ldots C_{jj}^{(n)}\ D_j^{(0)}\ldots D_j^{(n+d)}]$$

and where subscript j denotes jth row and subscript jj denotes the jth diagonal element. Note that from (2-6) the $C^{(j)}$ matrices are diagonal.

The parameter estimator in this case would involve ℓ regression vectors and ℓ covariance matrices, e.g., RLS-VFF2-MR in Appendix A. Let

$$\Theta = \mathrm{diag}[\theta_j']$$

$$H(t-1) = \mathrm{diag}[H_j(t+k_j-k-1)]$$

$$H_j(t+k_j-k-1) = \tilde{\theta}_j(t-d-1)-\tilde{\theta}_j(t+k_j-k-1)$$

$$\varepsilon'(t) = [\varepsilon_1(t+k_1-k)\,\varepsilon_2(t+k_2-k)\ldots\varepsilon_\ell(t+k_\ell-k)]$$

$$x(t-1)' = [x_1(t+k_1-k-1)\,x_2(t+k_2-k-1)\ldots x_\ell(t+k_\ell-k)]$$

Then the input and output error dynamics are described by

Lemma 4.7.

$$G(q^{-1}) = \begin{bmatrix} e(t) \\ \bar{v}(t-d-1) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & P(q^{-1}) \end{bmatrix} \begin{bmatrix} 0 \\ \varepsilon(t)+H(t-1)x(t-1) \end{bmatrix} \qquad (4\text{-}25)$$

where

$$G(q^{-1}) = \begin{bmatrix} A(q^{-1}) & -B(q^{-1}) \\ P(q^{-1}) & Q(q^{-1}) \end{bmatrix}$$

Proof: The upper part of (4-25) is simply the error form of model (2-16). In addition, from the control law and the parameter estimator, we obtain

$$P(q^{-1})[e(t)-\tilde{\theta}(t-d-1)x(t-1)]+Q(q^{-1})\bar{v}(t-d-1) = 0 \qquad (4\text{-}26)$$

and since

$$\varepsilon_j(t) = \theta_j'(t-1)x_j(t-1)$$

using the definition of $H(t-1)$, we can rewrite (4-26) as

$$P(q^{-1})[e(t)-\varepsilon(t)-H(t-1)x(t-1)]+Q(q^{-1})\bar{v}(t-d-1) = 0$$

from which the lower part of (4-25) follows.

Theorem 4.8. Subject to

(i) $K(q^{-1})$, $R(q^{-1})$ and n are known

(ii) $\det G(q^{-1}) \neq 0$ for $|z| \geq 1$

the STC (3-12) applied to a system described by (2-18) leads to

$$\lim_{t\to\infty} y(t) = y_s, \quad \lim_{t\to\infty} u(t) = u_s$$

while all process variables remain bounded.

Proof: From Lemma 4.1 and from (4-25), it follows that there exist positive constants $\bar{k}_1$, $\bar{k}_2$ such that

$$\| e'(t),\ \bar{v}'(t-d-1) \| \leq \bar{k}_1+\bar{k}_2 \max_{0\leq r\leq t} \| \varepsilon(r)+H(r-1)x(r-1) \| \qquad (4\text{-}27)$$

From the definition of $x(t-1)$, it follows that there exist positive constants $\bar{k}_3$, $\bar{k}_4$ for which

$$\| x(t-1) \| \leq \bar{k}_3+\bar{k}_4 \max_{0\leq r\leq t} \| e'(t),\ \bar{v}'(t-d-1) \|$$

This implies, in view of (4-27), property 5 of the estimators, and Lemma 4.2, that for the positive constants $k_1 = \bar{k}_3+\bar{k}_4\bar{k}_1$, $k_2 = \bar{k}_4\bar{k}_2$

$$\| x(t-1) \| \leqslant k_3+k_4 \max_{0\leqslant r\leqslant t} \| \varepsilon(r) \| \tag{4-28}$$

Now using the inequality

$$\sum_{i=1}^{\ell} \alpha_i \beta_i^{-1} \geqslant \left(\sum_{i=1}^{\ell} \alpha_i\right)\left(\sum_{i=1}^{\ell} \beta_i\right)^{-1} \text{ for } \alpha_i \geqslant 0,\ \beta_i > 0$$

properties 1 and 4 of the parameter estimators give that for $c_1 = \ell$, $c_2 = C$

$$\lim_{t\to\infty} \frac{\|\varepsilon(t)\|^2}{c_1+c_2\| x(t-1) \|^2} = 0$$

It thus follows from Lemma 4.3 that $\lim_{t\to\infty} \varepsilon(t) = 0$. Then from (4-28), one obtains that $x(t-1)$ is bounded and that $\lim_{t\to\infty}[\varepsilon(t)+H(t-1)x(t-1)] = 0$. This, in conjunction with (4-25) and Lemma 4.1, gives that $\lim_{t\to\infty} e(t) = 0$ and $\lim_{t\to\infty} \bar{v}(t-d-1) = 0$. Consequently, $y(t)$ tends to y_s (no offset) and $u(t)$ tends to u_s. This completes the proof.

Self-Tuning Controllers Based on the Right Interactor

The system is assumed to be of the form (3-2) and the control law is given by (3-18). Since $C(q^{-1})$ is not necessarily diagonal, in this case the estimation scheme is based on

$$y(t) = \Theta x(t-1)$$

where

$$\Theta = [C^{(0)} \ldots C^{(n)}\ D^{(0)} \ldots D^{(n+d)}]$$

$$x'(t-1) = [y'(t-d-1) \ldots y'(t-d-n-1)\ v'(t-d-1) \ldots\ v'(t-d-n-1)]$$

Such an estimation scheme involves one regression vector and one covariance matrix, e.g., the scheme RLSVFF2 in Appendix A. The equation which describes the error dynamics is

$$G(q^{-1}) = \begin{bmatrix} e(t) \\ \bar{v}(t-d) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & P(q^{-1}) \end{bmatrix} \begin{bmatrix} 0 \\ \varepsilon(t)+H(t-1)x(t-1) \end{bmatrix}$$

where $G(q^{-1})$ is as in Lemma 4.7. Then Theorem 4.8 applies mutatis mutandis and gives that $\lim_{t\to\infty} e(t) = 0 = \lim_{t\to\infty} \bar{v}(t)$. Consequently, assuming that the right interactor is known, Theorem 3.10 (ii) gives $\lim_{t\to\infty} u(t) = u_s$.

Special Cases

1. The single input-single output STC of Clarke and Gawthrop (21). In this case, there is one delay, k, present and the delay factorization is trivial, i.e., $K(q^{-1}) = q^{-k}$ and $R(q^{-1}) = 1$. The stability condition becomes

   $$P(q^{-1})B(q^{-1})+Q(q^{-1})A(q^{-1}) \neq 0 \text{ for } |q| \geqslant 1$$

   where $P(q^{-1})$, $B(q^{-1})$, $Q(q^{-1})$, and $A(q^{-1})$ are scalar polynomials. A detailed analysis of the algorithm can be found in (55).
2. The multivariable STC of Koivo (17). In this case, $K(q^{-1}) = q^{-k}\, I$ and $R(q^{-1}) = I$. Note that since $K(q^{-1}) = q^{-k}\, I$, one does not need to start from an $A(q^{-1})$-diagonal parametrization. Again, if P and Q are chosen such that G is stably invertible, the algorithm is asymptotically stable.

3. The case that $P(q^{-1})$ and $Q(q^{-1})$ commute. The stability condition then becomes, by using Schur's formula (56) for the evaluation of partitioned determinants,

$$\det[A(q^{-1})Q(q^{-1})+B(q^{-1})P(q^{-1})] \neq 0 \text{ for } |q| \geqslant 1$$

i.e., AQ+BP should have no latent root outside the open unit circle.

CHAPTER V
CONCLUSION

This work has addressed the problem of discrete-time control of multivariable plants. Convergent adaptive algorithms have been introduced which can treat open loop unstable or nonminimum phase systems. The factorization of the invariant, under feedback control, part of a transfer matrix led to the design of optimal Internal Model Controllers and disturbance estimators. A number of problems have not been treated in this work and may be challenges for further research. Stochastic considerations and influences of time varying parameters have not been treated. Nonlinearities were not included in the proposed adaptive schemes. Although the design of many nonlinear adaptive controllers may retain the essential simplicity of the linear case the stability issue is still open. The stability of indirect adaptive controllers is a subject of current research also. Finally, stable and causal inversion remains a challenge for many nonlinear systems.

APPENDIX A
PARAMETER ESTIMATORS

There are several methods for estimating the parameter vector θ, for example, the projection method (Goodwin et al., (15)) given in A.1. However, due to their computational simplicity and speed of convergence, recursive least squares (RLS) estimators are by far the most popular. Four types of RLS estimators are commonly used for STCs, namely:

1) RLS "standard" algorithm (RLS-SA). The equations for this algorithm are given in A.2. They include a modification first given by Goodwin et al. (15), which ensures that the matrix $[\hat{B}^0(t)+Q(0)]$ is always nonsingular, i.e., that the control law is solvable for u(t).
2) RLS with variable forgetting factor (RLS-VFF1) (Kershenbaum and Ydstie (57); Ydstie and Sargent (38)). By forgetting past data, this algorithm allows the estimates to adapt faster to changes in the plant parameters due to load or set point changes. But the forgetting ceases when close to steady state, thus limiting burst phenomena. The algorithm is described in A.3.
3) Approximate RLS with variable forgetting factor (RLS-VFF2) (Fortescue, Kershenbaum, and Ydstie (58); Cordero and Mayne (37)). This is an approximation of the previous algorithm which requires fewer calculations without sacrificing performance. The algorithm equations are given in A.4.

4) RLS multiple regression (RLS-MR) applicable to cases where more than one regression vector is present. Such an algorithm is given in A.5.

A.1 Projection Method

1) $\varepsilon(t) = w(t) - \hat{\theta}(t-1)x(t-1)$

2) $g(t-1) = x'(t-1)x(t-1)$

3) $\hat{\theta}(t) = \hat{\theta}(t-1) + \alpha(t)\,\dfrac{\varepsilon(t)x'(t-1)}{1+g(t-1)}$, $0<\alpha_1<\alpha(t)<\alpha_2<2$

(normally $\alpha(t) = 1$)

4) If $\hat{D}^{(0)}+(P(0)^{-1}Q(0)$ is singular, change $\alpha(t)$ and go to step 2.

A.2 RLS-SA

1) $\varepsilon(t) = w(t) - \hat{\theta}(t-1)x(t-1)$

2) $g(t-1) = \alpha(t)x'(t-1)F(t-1)x(t-1)$, $\alpha(t)$ is as in the projection method

3) $\hat{\theta}(t) = \hat{\theta}(t-1) + \alpha(t)\,\dfrac{\varepsilon(t)x'(t-1)F(t-1)}{1+g(t-1)}$

4) If $[\hat{D}_0+(P(0))^{-1}Q(0)]$ is singular, change $\alpha(t)$ and go to step 2.

5) $F(t) = [I-\alpha(t)\,\dfrac{F(t-1)x(t-1)x'(t-1)}{1+g(t-1)}]\;F(t-1)$

A.3 RLS-VFF1

1) as in RLS-SA

2) $p(t) = 1-g(t-1)-\alpha(t)\,\dfrac{\varepsilon'(t)K\,\varepsilon(t)}{N_0}$

where $N_0 > 0$, K = pos. def. matrix

4) $\lambda(t) = [p(t)+ (p(t)^2+4g(t-1))^{1/2}]/2$

5) $\hat{\theta}(t) = \hat{\theta}(t-1)+\alpha(t)\,\dfrac{\varepsilon(t)x'(t-1)F(t-1)}{\lambda(t)+g(t-1)}$

6) as in step 4 of the RLS-SA

7) $F(t) = [I-\alpha(t)\,\dfrac{F(t-1)x(t-1)x'(t-1)}{\lambda(t)+g(t-1)}]\;F(t-1)/\lambda(t)$

A.4 RLS-VFF2

1-4) as in RLS-SA

5) $W(t) = [I-\alpha(t)\ \frac{F(t-1)x(t-1)x'(t-1)}{1+g(t-1)}]\ F(t-1)$

6) $\lambda(t) = \max\ [\bar{\lambda}(t),\ \lambda_{min},\ \text{trace}\ (W(t))/\bar{F}]$

where $\bar{\lambda}(t) = 1 - \frac{\varepsilon'(t)K\ \varepsilon(t)}{N_1(1+g(t-1))}$, $N_1 > 0$, K = pos. def. matrix,

$\bar{F} > \text{trace}\ (F(0))$,

$0 < \lambda_{min} < 1$

7) $F(t) = W(t)/\lambda(t)$

A.5 RLS-VFF2-MR

1) $\varepsilon_j(t) = y(t)-x_j'(t-1)\hat{\theta}_j(t-1)$

2) $g_j(t-1) = \alpha(t)x_j'(t-1)F_j(t-1)x_j(t-1)$

3) $\hat{\theta}_j(t) = \hat{\theta}_j(t-1)+\alpha(t)\ \frac{F_j(t-1)x_j(t-1)}{1+g_j(t-1)}\ \varepsilon_j(t)$

4) same as in RLS-SA

5-7) same as in RLS-VFF2 for each $\lambda_j(t)$ and for each covariance $F_j(t)$ corresponding to $x_j(t-1)$.

APPENDIX B
THE INTERACTOR MATRIX

The following establishes the existence of the left interactor matrix $\xi(z)$ in (8).

First we note that given a nonsingular and strictly proper transfer function, $G(z)$, there exist unique integers k_i, $i = 1,2,\ldots,\ell$ such that

$$\lim_{z\to\infty} z^{k_i} G_i(z) = \tau_i; \quad G_i(z) = \text{ith row of } G(z)$$

is both finite and nonzero.

We define the first row $\xi(z)_1$ of $\xi(z)$ by

$$\xi(z)_1 = (z^{k_1}\ 0\ldots 0)$$

so that

$$\lim_{z\to\infty} \xi(z)_1\, G(z) = \tau_1 = \xi_1$$

If τ_2 is linearly independent from $\tau_1 = \xi_1$, then we get

$$\xi(z)_2 = (0\ z^{k_2}\ 0\ldots 0)$$

so that

$$\lim_{z\to\infty} \xi(z)_2\, G(z) = \tau_2$$

On the other hand, if τ_2 and ξ_1 are linearly dependent so that $\tau_2 = \alpha_1^1 \xi_1$, with $\alpha_1^1 \neq 0$, we let

$$\tilde{\xi}^1(z)_2 = z^{k_2^1}[(0, z^{k_2}\ 0\ldots0)-\alpha_1^1\xi(z)_1]$$

where k_2 is the unique integer for which

$$\lim_{z\to\infty} \tilde{\xi}^1(z)_2\ G(z) = \tilde{\xi}_2^1$$

is both finite and nonzero. If $\tilde{\xi}_2^1$ is linearly independent of ξ_1, we get

$$\xi(z)_2 = \tilde{\xi}^1(z)_2$$

and note that

$$\lim_{z\to\infty} \xi(z)_2\ G(z) = \tilde{\xi}_2^1$$

is linearly independent of ξ_1. If not, then

$$\tilde{\xi}_2^1 = \alpha_1^2\xi_1$$

and we let

$$\tilde{\xi}^2(z)_2 = z^{k_2^2}[\xi^1(z)_2-\alpha_1^2\xi(z)_1]$$

where k_2^2 is the unique integer for which

$$\lim_{z\to\infty} \tilde{\xi}^2(z)_2\ G(z) = \tilde{\xi}_2^2$$

is both finite and nonzero. If $\tilde{\xi}_2^2$ and ξ_1 are linearly independent, we get

$$\xi(z)_2 = \tilde{\xi}^2(z)_2$$

and if not, we repeat the procedure until linear independence is obtained. The procedure must terminate since det $G(z) \neq 0$ almost all z and since $k_i \geqslant 1$, $i = 1,2,\ldots,\ell$. The remaining rows of $\xi(z)$ are obtained in an analogous fashion. Finally, uniqueness of (z) can be establizhed as in Wolowich and Falb (8).

APPENDIX C
PROOF OF EQUATIONS (2-6)

Equations (2-6) will be shown for any positive integer p in place of k or d by use of induction. The proof given here corresponds to model (2-16). Model (2-3) is a subcase of model (2-16).

Step 1. For p = 0, trivial since the model (2-18) results. For p = 1, equations (2-6) are easily verified by substituting w(t+1) in the right hand side of (2-18), via (2-18) for p = 0.

Step 2. Assume that (2-19) and (2-6) are true up to p = p.

Step 3. It will be proved that (2-19) and (2-6) are true for p = p+1.

From (2-18):

$$w(t+p+1) = s+A^{(0)}w(t+p)+\ldots+A^{(n)}w(t+p-n)$$

$$+B^{(0)}v(t+p-k_{\ell}-r_m)+\ldots+B^{(n)}v(t+p-k_{\ell}-r_m-n)$$

But from step 2 w(t+p), w(t+p-1)...w(t+1) can be substituted via (2-19). The result is:

$$w(t+p+1) = s+A^{(0)}[R_p s + \sum_{i=0}^{n} C_p^{(i)} w(t-i) + \sum_{i=0}^{n} D^{(i)} v(t+p-1-r_m-k_\ell-i)]$$

$$+ A^{(1)}[R_{p-1}s+ \sum_{i=0}^{n} C_{p-1}^{(i)} w(t-i) + \sum_{i=0}^{n+p-2} D_{p-1}^{(i)} v(t+p-2-r_m-k_\ell-i)]$$

$$+\dots+ A^{(p-1)}[R_1 s+ \sum_{i=0}^{n} C_1^{(i)} w(t-i) + \sum_{i=0}^{n} D_1^{(i)} v(t-r_m-k_\ell-i)]$$

$$+ A^{(p)} w(t) + A^{(p+1)} w(t-1) + \dots + A^{(n)} W(t+p-n) +$$

$$+ B^{(0)} v(t+p-k_\ell-r_m) + B^{(1)} v(t+p-k_\ell-r_m-1)$$

$$+\dots+B^{(n)} v(t+o-k_\ell-r_m-n)$$

or

$$w(t+p+1) = [I+ \sum_{k=0}^{p-1} A^{(k)} R_{p-k}]s+[\sum_{k=0}^{p-1} A^{(k)} C_{p-k}^{(0)} + A^{(p)}]w(t)$$

$$+\dots+ [\sum_{k=0}^{p-1} A^{(k)} C_{p-k}^{(i)} + A^{(p+i)}]w(t-i)$$

$$+\dots+ [\sum_{k=0}^{p-1} A^{(k)} C_{p-k}^{(n-p)} + A^{(n)}]w(t+p-n) +$$

$$[\sum_{k=0}^{p-1} A^{(k)} C_{p-k}^{(n-p+1)}]w(t+p-n-1) +$$

$$+\dots+ [\sum_{k=0}^{p-1} A^{(k)} C_{p-k}^{(n)}]w(t-n) +$$

$$+ B^{(0)} v(t+p-k_\ell-r_m) + [A^{(0)} D_p^{(0)} + B^{(1)}]v(t+p-k_\ell-r_m-1) +$$

$$+ [A^{(0)} D_p^{(1)} + A^{(1)} D_{p-1}^{(0)} + B^{(2)}]v(t+p-k_\ell-r_m-2) + \dots +$$

$$+ [A^{(0)}D_p^{(n-1)}+A^{(1)}D_{p-1}^{(n-2)}$$

$$+\dots+A^{(p-1)}D_1^{(n-p)}+B^{(n)}]v(t+p-k_\ell-r_m-n) +$$

$$+ [A^{(0)}D_p^{(n)}+A^{(1)}D_{p-1}^{(n-1)}$$

$$+\dots+A^{(p-1)}D_1^{(n-p+1)}]v(t+p-k_\ell-r_m-n-1) +$$

$$+ [A^{(0)}D_p^{(n+p-1)}+A^{(1)}D_{p-1}^{(n+p-2)}$$

$$+\dots+A^{(p-1)}D_1^{(n)}]v(t-k_\ell-r_m-n)$$

from which (2-6) follows.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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